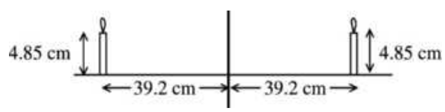
34

GEOMETRIC OPTICS

34.1. **IDENTIFY** and **SET UP:** Plane mirror: $s = -s'$ (Eq. 34.1) and $m = y'/y = -s'/s = +1$ (Eq. 34.2). We are given s and y and are asked to find s' and y'.

EXECUTE: The object and image are shown in Figure 34.1.



$s' = -s = -39.2$ cm

$|y'| = |m||y| = (+1)(4.85 \text{ cm})$

$|y'| = 4.85$ cm

Figure 34.1

The image is 39.2 cm to the right of the mirror and is 4.85 cm tall.

EVALUATE: For a plane mirror the image is always the same distance behind the mirror as the object is in front of the mirror. The image always has the same height as the object.

34.2. **IDENTIFY:** Similar triangles say $\dfrac{h_{\text{tree}}}{h_{\text{mirror}}} = \dfrac{d_{\text{tree}}}{d_{\text{mirror}}}$.

SET UP: $d_{\text{mirror}} = 0.350$ m, $h_{\text{mirror}} = 0.0400$ m and $d_{\text{tree}} = 28.0 \text{ m} + 0.350$ m.

EXECUTE: $h_{\text{tree}} = h_{\text{mirror}} \dfrac{d_{\text{tree}}}{d_{\text{mirror}}} = 0.040 \text{ m} \dfrac{28.0 \text{ m} + 0.350 \text{ m}}{0.350 \text{ m}} = 3.24$ m.

EVALUATE: The image of the tree formed by the mirror is 28.0 m behind the mirror and is 3.24 m tall.

34.3. **IDENTIFY** and **SET UP:** The virtual image formed by a plane mirror is the same size as the object and the same distance from the mirror as the object.

EXECUTE: $s' = -s$. The image of the tip is 12.0 cm behind the mirror surface and the image of the end of the eraser is 21.0 cm behind the mirror surface. The length of the image is 9.0 cm, the same as the length of the object. The image of the tip of the lead is the closest to the mirror surface.

EVALUATE: The same result would hold no matter how far the pencil was from the mirror.

34.4. **IDENTIFY:** $f = R/2$

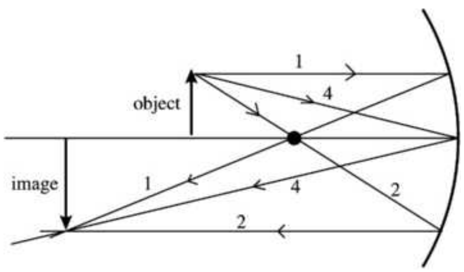
SET UP: For a concave mirror $R > 0$.

EXECUTE: **(a)** $f = \dfrac{R}{2} = \dfrac{34.0 \text{ cm}}{2} = 17.0$ cm

EVALUATE: **(b)** The image formation by the mirror is determined by the law of reflection and that is unaffected by the medium in which the light is traveling. The focal length remains 17.0 cm.

34.5. **IDENTIFY** and **SET UP:** Use Eq. (34.6) to calculate s' and use Eq. (34.7) to calculate y'. The image is real if s' is positive and is erect if $m > 0$. Concave means R and f are positive, $R = +22.0$ cm; $f = R/2 = +11.0$ cm.

EXECUTE: (a)



Three principal rays, numbered as in Section 34.2, are shown in Figure 34.5. The principal ray diagram shows that the image is real, inverted, and enlarged.

Figure 34.5

(b) $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$

$\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}=\frac{s-f}{sf}$ so $s'=\frac{sf}{s-f}=\frac{(16.5\text{ cm})(11.0\text{ cm})}{16.5\text{ cm}-11.0\text{ cm}}=+33.0\text{ cm}$

$s'>0$ so real image, 33.0 cm to left of mirror vertex

$m=-\frac{s'}{s}=-\frac{33.0\text{ cm}}{16.5\text{ cm}}=-2.00$ ($m<0$ means inverted image) $|y'|=|m||y|=2.00(0.600\text{ cm})=1.20\text{ cm}$

EVALUATE: The image is 33.0 cm to the left of the mirror vertex. It is real, inverted, and is 1.20 cm tall (enlarged). The calculation agrees with the image characterization from the principal ray diagram. A concave mirror used alone always forms a real, inverted image if $s>f$ and the image is enlarged if $f<s<2f$.

34.6. **IDENTIFY:** Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ and $m=-\frac{s'}{s}$.

SET UP: For a convex mirror, $R<0$. $R=-22.0\text{ cm}$ and $f=\frac{R}{2}=-11.0\text{ cm}$.

EXECUTE: **(a)** The principal-ray diagram is sketched in Figure 34.6.

(b) $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$. $s'=\frac{sf}{s-f}=\frac{(16.5\text{ cm})(-11.0\text{ cm})}{16.5\text{ cm}-(-11.0\text{ cm})}=-6.6\text{ cm}$. $m=-\frac{s'}{s}=-\frac{-6.6\text{ cm}}{16.5\text{ cm}}=+0.400$.

$|y'|=|m||y|=(0.400)(0.600\text{ cm})=0.240\text{ cm}$. The image is 6.6 cm to the right of the mirror. It is 0.240 cm tall. $s'<0$, so the image is virtual. $m>0$, so the image is erect.

EVALUATE: The calculated image properties agree with the image characterization from the principal-ray diagram.

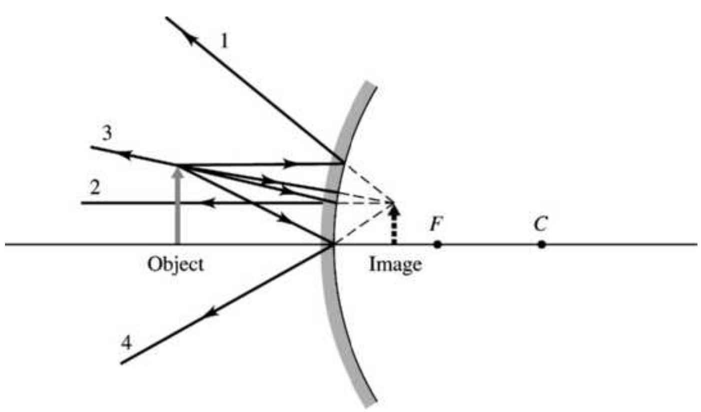


Figure 34.6

34.7. **IDENTIFY:** $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$. $m=-\frac{s'}{s}$. $|m|=\frac{|y'|}{y}$. Find m and calculate y'.

SET UP: $f=+1.75$ m.

EXECUTE: $s \gg f$ so $s'=f=1.75$ m.

$$m=-\frac{s'}{s}=-\frac{1.75\text{ m}}{5.58\times10^{10}\text{ m}}=-3.14\times10^{-11}.$$

$|y'|=|m||y|=(3.14\times10^{-11})(6.794\times10^{6}\text{ m})=2.13\times10^{-4}\text{ m}=0.213$ mm.

EVALUATE: The image is real and is 1.75 m in front of the mirror.

34.8. **IDENTIFY:** Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ and $m=-\frac{s'}{s}$.

SET UP: The mirror surface is convex so $R=-3.00$ cm. $s=24.0\text{ cm}-3.00\text{ cm}=21.0$ cm.

EXECUTE: $f=\frac{R}{2}=-1.50$ cm. $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$. $s'=\frac{sf}{s-f}=\frac{(21.0\text{ cm})(-1.50\text{ cm})}{21.0\text{ cm}-(-1.50\text{ cm})}=-1.40$ cm. The image is 1.40 cm behind the surface so it is $3.00\text{ cm}-1.40\text{ cm}=1.60$ cm from the center of the ornament, on the same side as the object. $m=-\frac{s'}{s}=-\frac{-1.40\text{ cm}}{21.0\text{ cm}}=+0.0667$. $|y'|=|m||y|=(0.0667)(3.80\text{ mm})=0.253$ mm.

EVALUATE: The image is virtual, upright and smaller than the object.

34.9. **IDENTIFY:** The shell behaves as a spherical mirror.

SET UP: The equation relating the object and image distances to the focal length of a spherical mirror is $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$, and its magnification is given by $m=-\frac{s'}{s}$.

EXECUTE: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{s}=\frac{2}{-18.0\text{ cm}}-\frac{1}{-6.00\text{ cm}}\Rightarrow s=18.0$ cm from the vertex.

$m=-\frac{s'}{s}=-\frac{-6.00\text{ cm}}{18.0\text{ cm}}=\frac{1}{3}\Rightarrow y'=\frac{1}{3}(1.5\text{ cm})=0.50$ cm. The image is 0.50 cm tall, erect and virtual.

EVALUATE: Since the magnification is less than one, the image is smaller than the object.

34.10. **IDENTIFY:** The bottom surface of the bowl behaves as a spherical convex mirror.

SET UP: The equation relating the object and image distances to the focal length of a spherical mirror is $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$, and its magnification is given by $m=-\frac{s'}{s}$.

EXECUTE: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{s'}=\frac{-2}{35\text{ cm}}-\frac{1}{90\text{ cm}}\Rightarrow s'=-15$ cm behind the bowl.

$m=-\frac{s'}{s}=\frac{15\text{ cm}}{90\text{ cm}}=0.167\Rightarrow y'=(0.167)(2.0\text{ cm})=0.33$ cm. The image is 0.33 cm tall, erect and virtual.

EVALUATE: Since the magnification is less than one, the image is smaller than the object.

34.11. **IDENTIFY:** Express the lateral magnification of a mirror in terms of its focal length and the object distance and then make use of the result.

SET UP: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$. $m=-\frac{s'}{s}$.

EXECUTE: **(a)** Using $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$, we have $\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}=\frac{s-f}{sf}$. $s'=\frac{sf}{s-f}$. The lateral magnification is

$$m=-\frac{s'}{s}=-\frac{f}{s-f}=\frac{f}{f-s}.$$

(b) $m=\pm1$. For $m=+1$, $f=f-s$ and $s=0$. This solution is excluded in the statement of the problem. For $m=-1$, $f=-(f-s)$ and $s=2f=28.0$ cm. The object is 28.0 cm from the mirror vertex. Negative m means the image is inverted.

(c) For a convex mirror all images are virtual and erect, so $m = +\frac{1}{2}$. $\frac{f}{f-s} = \frac{1}{2}$. $2f = f - s$ and $s = -f = +8.00$ cm. The object is 8.00 cm from the mirror vertex. Positive m means the image is erect.

EVALUATE: The sign of f can vary, depending on the type of mirror.

34.12. **IDENTIFY:** In part (a), the shell is a concave mirror, but in (b) it is a convex mirror. The magnitude of its focal length is the same in both cases, but the sign reverses.

SET UP: For the orientation of the shell shown in the figure in the problem, $R = +12.0$ cm. When the glass is reversed, so the seed faces a convex surface, $R = -12.0$ cm. $\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$ and $m = \frac{y'}{y} = -\frac{s'}{s}$.

EXECUTE: **(a)** $R = +12.0$ cm. $\frac{1}{s'} = \frac{2}{R} - \frac{1}{s} = \frac{2s - R}{Rs}$ and $s' = \frac{Rs}{2s - R} = \frac{(12.0\text{ cm})(15.0\text{ cm})}{30.0\text{ cm} - 12.0\text{ cm}} = +10.0$ cm.

$m = -\frac{s'}{s} = -\frac{10.0\text{ cm}}{15.0\text{ cm}} = -0.667$. $y' = my = -2.20$ mm. The image is 10.0 cm to the left of the shell vertex and is 2.20 mm tall.

(b) $R = -12.0$ cm. $s' = \frac{(-12.0\text{ cm})(15.0\text{ cm})}{30.0\text{ cm} + 12.0\text{ cm}} = -4.29$ cm. $m = -\frac{-4.29\text{ cm}}{15.0\text{ cm}} = +0.286$.

$y' = my = 0.944$ mm. The image is 4.29 cm to the right of the shell vertex and is 0.944 mm tall.

EVALUATE: In (a), $s > R/2$ and the mirror is concave, so the image is real. In (b) the image is virtual because a convex mirror always forms a virtual image.

34.13. **IDENTIFY:** $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ and $m = \frac{y'}{y} = -\frac{s'}{s}$.

SET UP: $m = +2.00$ and $s = 1.25$ cm. An erect image must be virtual.

EXECUTE: **(a)** $s' = \frac{sf}{s-f}$ and $m = -\frac{f}{s-f}$. For a concave mirror, m can be larger than 1.00. For a convex mirror, $|f| = -f$ so $m = +\frac{|f|}{s+|f|}$ and m is always less than 1.00. The mirror must be concave $(f > 0)$.

(b) $\frac{1}{f} = \frac{s'+s}{ss'}$. $f = \frac{ss'}{s+s'}$. $m = -\frac{s'}{s} = +2.00$ and $s' = -2.00s$. $f = \frac{s(-2.00s)}{s - 2.00s} = +2.00s = +2.50$ cm.

$R = 2f = +5.00$ cm.

(c) The principal-ray diagram is drawn in Figure 34.13.

EVALUATE: The principal-ray diagram agrees with the description from the equations.

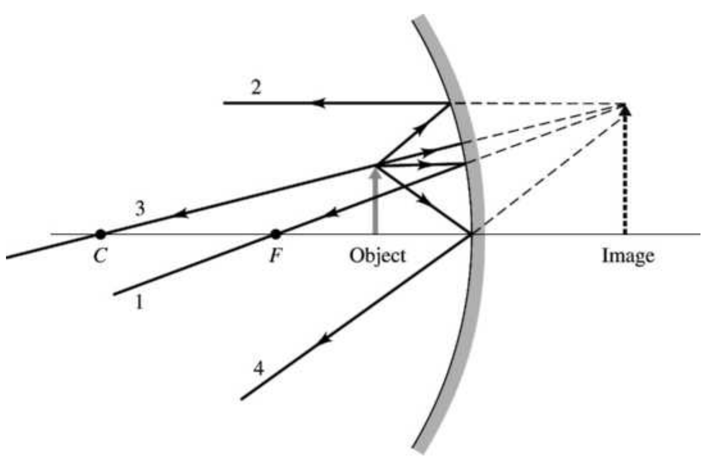


Figure 34.13

34.14. **IDENTIFY:** Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ and $m=-\frac{s'}{s}$.

SET UP: For a concave mirror, $R>0$. $R=32.0\text{ cm}$ and $f=\frac{R}{2}=16.0\text{ cm}$.

EXECUTE: **(a)** $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$. $s'=\frac{sf}{s-f}=\frac{(12.0\text{ cm})(16.0\text{ cm})}{12.0\text{ cm}-16.0\text{ cm}}=-48.0\text{ cm}$. $m=-\frac{s'}{s}=-\frac{-48.0\text{ cm}}{12.0\text{ cm}}=+4.00$.

(b) $s'=-48.0\text{ cm}$, so the image is 48.0 cm to the right of the mirror. $s'<0$ so the image is virtual.

(c) The principal-ray diagram is sketched in Figure 34.14. The rules for principal rays apply only to paraxial rays. Principal ray 2, that travels to the mirror along a line that passes through the focus, makes a large angle with the optic axis and is not described well by the paraxial approximation. Therefore, principal ray 2 is not included in the sketch.

EVALUATE: A concave mirror forms a virtual image whenever $s<f$.

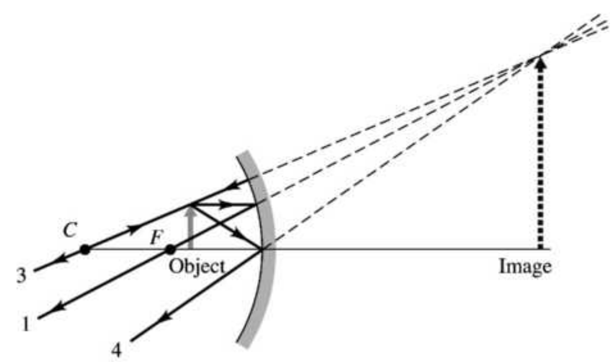
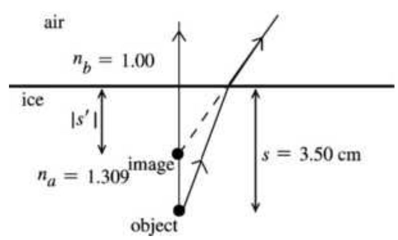


Figure 34.14

34.15. **IDENTIFY:** Apply Eq. (34.11), with $R\to\infty$. $|s'|$ is the apparent depth.

SET UP: The image and object are shown in Figure 34.15.



$\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$;

$R\to\infty$ (flat surface), so

$\frac{n_a}{s}+\frac{n_b}{s'}=0$

Figure 34.15

EXECUTE: $s'=-\frac{n_b s}{n_a}=-\frac{(1.00)(3.50\text{ cm})}{1.309}=-2.67\text{ cm}$

The apparent depth is 2.67 cm.

EVALUATE: When the light goes from ice to air (larger to smaller n), it is bent away from the normal and the virtual image is closer to the surface than the object is.

34.16. **IDENTIFY:** The surface is flat so $R\to\infty$ and $\frac{n_a}{s}+\frac{n_b}{s'}=0$.

SET UP: The light travels from the fish to the eye, so $n_a=1.333$ and $n_b=1.00$. When the fish is viewed, $s=7.0\text{ cm}$. The fish is $20.0\text{ cm}-7.0\text{ cm}=13.0\text{ cm}$ above the mirror, so the image of the fish is 13.0 cm below the mirror and $20.0\text{ cm}+13.0\text{ cm}=33.0\text{ cm}$ below the surface of the water. When the image is viewed, $s=33.0\text{ cm}$.

EXECUTE: **(a)** $s' = -\left(\frac{n_b}{n_a}\right)s = -\left(\frac{1.00}{1.333}\right)(7.0\text{ cm}) = -5.25\text{ cm}$. The apparent depth is 5.25 cm.

(b) $s' = -\left(\frac{n_b}{n_a}\right)s = -\left(\frac{1.00}{1.333}\right)(33.0\text{ cm}) = -24.8\text{ cm}$. The apparent depth of the image of the fish in the mirror is 24.8 cm.

EVALUATE: In each case the apparent depth is less than the actual depth of what is being viewed.

34.17. **IDENTIFY:** Think of the surface of the water as a section of a sphere having an infinite radius of curvature.

SET UP: $\frac{n_a}{s} + \frac{n_b}{s'} = 0$. $n_a = 1.00$. $n_b = 1.333$.

EXECUTE: The image is $5.20\text{ m} - 0.80\text{ m} = 4.40\text{ m}$ above the surface of the water, so $s' = -4.40$ m.

$s = -\frac{n_a}{n_b}s' = -\left(\frac{1.00}{1.333}\right)(-4.40\text{ m}) = +3.30\text{ m}$.

EVALUATE: The diving board is closer to the water than it looks to the swimmer.

34.18. **IDENTIFY:** Think of the surface of the water as a section of a sphere having an infinite radius of curvature.

SET UP: $\frac{n_a}{s} + \frac{n_b}{s'} = 0$. $n_a = 1.333$. $n_b = 1.00$.

EXECUTE: The image is 5.00 m below surface of the water, so $s' = -5.00$ m.

$s = -\frac{n_a}{n_b}s' = -\left(\frac{1.333}{1.00}\right)(-5.00\text{ m}) = 6.66\text{ m}$.

EVALUATE: The water is deeper than it appears to the person.

34.19. **IDENTIFY:** $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$. $m = -\frac{n_a s'}{n_b s}$. Light comes from the fish to the person's eye.

SET UP: $R = -14.0$ cm. $s = +14.0$ cm. $n_a = 1.333$ (water). $n_b = 1.00$ (air). Figure 34.19 shows the object and the refracting surface.

EXECUTE: **(a)** $\frac{1.333}{14.0\text{ cm}} + \frac{1.00}{s'} = \frac{1.00 - 1.333}{-14.0\text{ cm}}$. $s' = -14.0$ cm. $m = -\frac{(1.333)(-14.0\text{ cm})}{(1.00)(14.0\text{ cm})} = +1.33$.

The fish's image is 14.0 cm to the left of the bowl surface so is at the center of the bowl and the magnification is 1.33.

(b) The focal point is at the image location when $s \to \infty$. $\frac{n_b}{s'} = \frac{n_b - n_a}{R}$. $n_a = 1.00$. $n_b = 1.333$.

$R = +14.0$ cm. $\frac{1.333}{s'} = \frac{1.333 - 1.00}{14.0\text{ cm}}$. $s' = +56.0$ cm. s' is greater than the diameter of the bowl, so the surface facing the sunlight does not focus the sunlight to a point inside the bowl. The focal point is outside the bowl and there is no danger to the fish.

EVALUATE: In part (b) the rays refract when they exit the bowl back into the air so the image we calculated is not the final image.

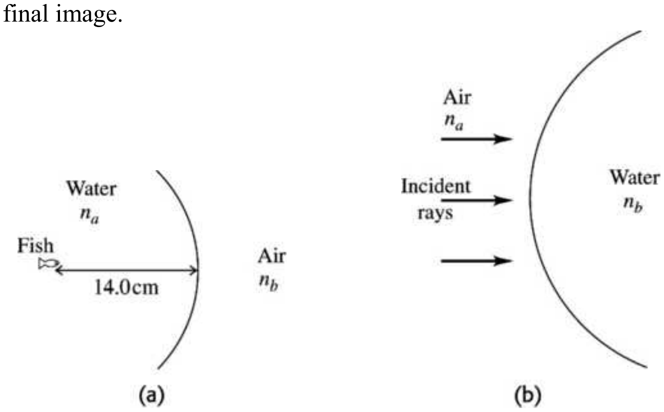


Figure 34.19

34.20. **IDENTIFY:** Apply $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$.

SET UP: For a convex surface, $R>0$. $R=+3.00$ cm. $n_a=1.00$, $n_b=1.60$.

EXECUTE: **(a)** $s\to\infty$. $\frac{n_b}{s'}=\frac{n_b-n_a}{R}$. $s'=\left(\frac{n_b}{n_b-n_a}\right)R=\left(\frac{1.60}{1.60-1.00}\right)(+3.00\text{ cm})=+8.00$ cm. The image is 8.00 cm to the right of the vertex.

(b) $s=12.0$ cm. $\frac{1.00}{12.0\text{ cm}}+\frac{1.60}{s'}=\frac{1.60-1.00}{3.00\text{ cm}}$. $s'=+13.7$ cm. The image is 13.7 cm to the right of the vertex.

(c) $s=2.00$ cm. $\frac{1.00}{2.00\text{ cm}}+\frac{1.60}{s'}=\frac{1.60-1.00}{3.00\text{ cm}}$. $s'=-5.33$ cm. The image is 5.33 cm to the left of the vertex.

EVALUATE: The image can be either real $(s'>0)$ or virtual $(s'<0)$, depending on the distance of the object from the refracting surface.

34.21. **IDENTIFY:** The hemispherical glass surface forms an image by refraction. The location of this image depends on the curvature of the surface and the indices of refraction of the glass and oil.

SET UP: The image and object distances are related to the indices of refraction and the radius of curvature by the equation $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$.

EXECUTE: $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}\Rightarrow\frac{1.45}{s}+\frac{1.60}{1.20\text{ m}}=\frac{0.15}{0.0300\text{ m}}\Rightarrow s=39.5$ cm

EVALUATE: The presence of the oil changes the location of the image.

34.22. **IDENTIFY:** $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$. $m=-\frac{n_a s'}{n_b s}$.

SET UP: $R=+4.00$ cm. $n_a=1.00$. $n_b=1.60$. $s=24.0$ cm.

EXECUTE: $\frac{1}{24.0\text{ cm}}+\frac{1.60}{s'}=\frac{1.60-1.00}{4.00\text{ cm}}$. $s'=+14.8$ cm. $m=-\frac{(1.00)(14.8\text{ cm})}{(1.60)(24.0\text{ cm})}=-0.385$.

$|y'|=|m||y|=(0.385)(1.50\text{ mm})=0.578$ mm. The image is 14.8 cm to the right of the vertex and is 0.578 mm tall. $m<0$, so the image is inverted.

EVALUATE: The image is real.

34.23. **IDENTIFY:** Apply Eqs. (34.11) and (34.12). Calculate s' and y'. The image is erect if $m>0$.

SET UP: The object and refracting surface are shown in Figure 34.23.

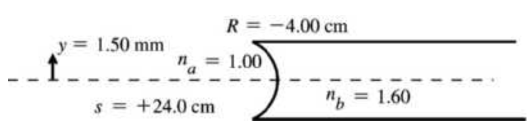


Figure 34.23

EXECUTE: $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$

$\frac{1.00}{24.0\text{ cm}}+\frac{1.60}{s'}=\frac{1.60-1.00}{-4.00\text{ cm}}$

Multiplying by 24.0 cm gives $1.00+\frac{38.4}{s'}=-3.60$

$\frac{38.4\text{ cm}}{s'}=-4.60$ and $s'=-\frac{38.4\text{ cm}}{4.60}=-8.35$ cm

Eq. (34.12): $m = -\frac{n_a s'}{n_b s} = -\frac{(1.00)(-8.35\text{ cm})}{(1.60)(+24.0\text{ cm})} = +0.217$

$|y'| = |m||y| = (0.217)(1.50\text{ mm}) = 0.326\text{ mm}$

EVALUATE: The image is virtual $(s' < 0)$ and is 8.35 cm to the left of the vertex. The image is erect $(m > 0)$ and is 0.326 mm tall. R is negative since the center of curvature of the surface is on the incoming side.

34.24. **IDENTIFY:** The hemispherical glass surface forms an image by refraction. The location of this image depends on the curvature of the surface and the indices of refraction of the glass and liquid.

SET UP: The image and object distances are related to the indices of refraction and the radius of curvature by the equation $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$.

EXECUTE: $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \Rightarrow \frac{n_a}{14.0\text{ cm}} + \frac{1.60}{-9.00\text{ cm}} = \frac{1.60 - n_a}{-4.00\text{ cm}} \Rightarrow n_a = 1.24.$

EVALUATE: The result is a reasonable refractive index for liquids.

34.25. **IDENTIFY:** Use $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ to calculate f. Then apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ and $m = \frac{y'}{y} = -\frac{s'}{s}$.

SET UP: $R_1 \to \infty$. $R_2 = -13.0\text{ cm}$. If the lens is reversed, $R_1 = +13.0\text{ cm}$ and $R_2 \to \infty$.

EXECUTE: **(a)** $\frac{1}{f} = (0.70)\left(\frac{1}{\infty} - \frac{1}{-13.0\text{ cm}}\right) = \frac{0.70}{13.0\text{ cm}}$ and $f = 18.6\text{ cm}$. $\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{s-f}{sf}$.

$s' = \frac{sf}{s-f} = \frac{(22.5\text{ cm})(18.6\text{ cm})}{22.5\text{ cm} - 18.6\text{ cm}} = 107\text{ cm}$. $m = -\frac{s'}{s} = -\frac{107\text{ cm}}{22.5\text{ cm}} = -4.76.$

$y' = my = (-4.76)(3.75\text{ mm}) = -17.8\text{ mm}$. The image is 107 cm to the right of the lens and is 17.8 mm tall. The image is real and inverted.

(b) $\frac{1}{f} = (n-1)\left(\frac{1}{13.0\text{ cm}} - \frac{1}{\infty}\right)$ and $f = 18.6\text{ cm}$. The image is the same as in part (a).

EVALUATE: Reversing a lens does not change the focal length of the lens.

34.26. **IDENTIFY:** $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$. The sign of f determines whether the lens is converging or diverging.

SET UP: $s = 16.0\text{ cm}$. $s' = -12.0\text{ cm}$.

EXECUTE: **(a)** $f = \frac{ss'}{s+s'} = \frac{(16.0\text{ cm})(-12.0\text{ cm})}{16.0\text{ cm} + (-12.0\text{ cm})} = -48.0\text{ cm}$. $f < 0$ and the lens is diverging.

(b) $m = -\frac{s'}{s} = -\frac{-12.0\text{ cm}}{16.0\text{ cm}} = +0.750$. $|y'| = |m|\,y = (0.750)(8.50\text{ mm}) = 6.38\text{ mm}$. $m > 0$ and the image is erect.

(c) The principal-ray diagram is sketched in Figure 34.26.

EVALUATE: A diverging lens always forms an image that is virtual, erect and reduced in size.

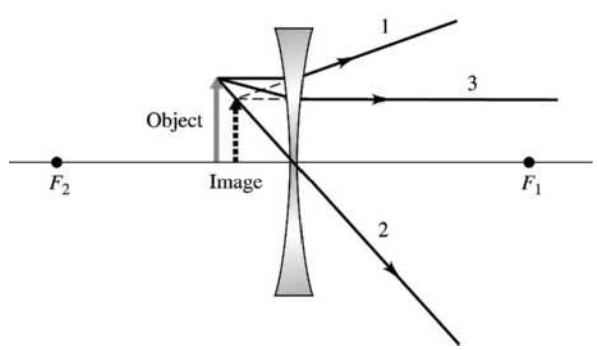


Figure 34.26

34.27. **IDENTIFY:** Use the lensmaker's equation to calculate f.

SET UP: The lensmaker's equation is $\frac{1}{s}+\frac{1}{s'}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$, and the magnification of the lens is $m=-\frac{s'}{s}$.

EXECUTE: **(a)** $\frac{1}{s}+\frac{1}{s'}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\Rightarrow\frac{1}{24.0\text{ cm}}+\frac{1}{s'}=(1.52-1)\left(\frac{1}{-7.00\text{ cm}}-\frac{1}{-4.00\text{ cm}}\right)$

$$\Rightarrow s'=71.2\text{ cm, to the right of the lens.}$$

(b) $m=-\frac{s'}{s}=-\frac{71.2\text{ cm}}{24.0\text{ cm}}=-2.97$

EVALUATE: Since the magnification is negative, the image is inverted.

34.28. **IDENTIFY:** Apply $m=\frac{y'}{y}=-\frac{s'}{s}$ to relate s' and s and then use $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$.

SET UP: Since the image is inverted, $y'<0$ and $m<0$.

EXECUTE: $m=\frac{y'}{y}=\frac{-4.50\text{ cm}}{3.20\text{ cm}}=-1.406$. $m=-\frac{s'}{s}$ gives $s'=+1.406s$. $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ gives $\frac{1}{s}+\frac{1}{1.406s}=\frac{1}{90.0\text{ cm}}$ and $s=154$ cm. $s'=(1.406)(154\text{ cm})=217$ cm. The object is 154 cm to the left of the lens. The image is 217 cm to the right of the lens and is real.

EVALUATE: For a single lens an inverted image is always real.

34.29. **IDENTIFY:** The thin-lens equation applies in this case.

SET UP: The thin-lens equation is $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$, and the magnification is $m=-\frac{s'}{s}=\frac{y'}{y}$.

EXECUTE: $m=\frac{y'}{y}=\frac{34.0\text{ mm}}{8.00\text{ mm}}=4.25=-\frac{s'}{s}=-\frac{-12.0\text{ cm}}{s}\Rightarrow s=2.82$ cm. The thin-lens equation gives $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow f=3.69$ cm.

EVALUATE: Since the focal length is positive, this is a converging lens. The image distance is negative because the object is inside the focal point of the lens.

34.30. **IDENTIFY:** Apply $m=-\frac{s'}{s}$ to relate s and s'. Then use $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$.

SET UP: Since the image is to the right of the lens, $s'>0$. $s'+s=6.00$ m.

EXECUTE: **(a)** $s'=80.0s$ and $s+s'=6.00$ m gives $81.00s=6.00$ m and $s=0.0741$ m. $s'=5.93$ m.

(b) The image is inverted since both the image and object are real $(s'>0, s>0)$.

(c) $\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}=\frac{1}{0.0741\text{ m}}+\frac{1}{5.93\text{ m}}\Rightarrow f=0.0732$ m, and the lens is converging.

EVALUATE: The object is close to the lens and the image is much farther from the lens. This is typical for slide projectors.

34.31. **IDENTIFY:** Apply $\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$.

SET UP: For a distant object the image is at the focal point of the lens. Therefore, $f=1.87$ cm. For the double-convex lens, $R_1=+R$ and $R_2=-R$, where $R=2.50$ cm.

EXECUTE: $\frac{1}{f}=(n-1)\left(\frac{1}{R}-\frac{1}{-R}\right)=\frac{2(n-1)}{R}$. $n=\frac{R}{2f}+1=\frac{2.50\text{ cm}}{2(1.87\text{ cm})}+1=1.67$.

EVALUATE: $f>0$ and the lens is converging. A double-convex lens is always converging.

34.32. **IDENTIFY:** Use the lensmaker's formula to find the radius of curvature of the lens of the eye.

SET UP: $\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$. If R is the radius of the lens, then $R_1=R$ and $R_2=-R$. $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$. $m=\frac{y'}{y}=-\frac{s'}{s}$.

EXECUTE: **(a)** $\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=(n-1)\left(\frac{1}{R}-\frac{1}{-R}\right)=\frac{2(n-1)}{R}$.
$R=2(n-1)f=2(0.44)(8.0\text{ mm})=7.0\text{ mm}$.

(b) $\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}=\frac{s-f}{sf}$. $s'=\frac{sf}{s-f}=\frac{(30.0\text{ cm})(0.80\text{ cm})}{30.0\text{ cm}-0.80\text{ cm}}=0.82\text{ cm}=8.2\text{ mm}$. The image is 8.2 mm from the lens, on the side opposite the object. $m=-\frac{s'}{s}=-\frac{0.82\text{ cm}}{30.0\text{ cm}}=-0.0273$.
$y'=my=(-0.0273)(16\text{ cm})=0.44\text{ cm}=4.4\text{ mm}$. $s'>0$ so the image is real. $m<0$ so the image is inverted.

EVALUATE: The lens is converging and has a very short focal length. As long as the object is farther than 7.0 mm from the eye, the lens forms a real image.

34.33. **IDENTIFY:** First use the lensmaker's formula to find the radius of curvature of the cornea.

SET UP: $\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$. $R_1=+5.0\text{ mm}$. $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$. $m=\frac{y'}{y}=-\frac{s'}{s}$.

EXECUTE: **(a)** $\frac{1}{f(n-1)}=\frac{1}{R_1}-\frac{1}{R_2}$. $\frac{1}{R_2}=\frac{1}{R_1}-\frac{1}{f(n-1)}=\frac{1}{+5.0\text{ mm}}-\frac{1}{(18.0\text{ mm})(0.38)}$ so $R_2=18.6\text{ mm}$.

(b) $\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}=\frac{s-f}{sf}$. $s'=\frac{sf}{s-f}=\frac{(25\text{ cm})(1.8\text{ cm})}{25\text{ cm}-1.8\text{ cm}}=1.9\text{ cm}=19\text{ mm}$.

(c) $m=-\frac{s'}{s}=-\frac{1.9\text{ cm}}{25\text{ cm}}=-0.076$. $y'=my=(-0.076)(8.0\text{ mm})=-0.61\text{ mm}$. $s'>0$ so the image is real. $m<0$ so the image is inverted.

EVALUATE: The cornea alone would focus an object at a distance of 19 mm, which is not at the retina. We must consider the effects of the lens of the eye and the fact that the eye is filled with liquid having an index of refraction.

34.34. **IDENTIFY:** We know where the image is formed and want to find where the object is.

SET UP: $m=\frac{y'}{y}=-\frac{s'}{s}$. Since the image is erect, $y'>0$ and $m>0$. $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$.

EXECUTE: $m=\frac{y'}{y}=\frac{1.30\text{ cm}}{0.400\text{ cm}}=+3.25$. $m=-\frac{s'}{s}=+3.25$ gives $s'=-3.25s$. $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ gives $\frac{1}{s}+\frac{1}{-3.25s}=\frac{1}{7.00\text{ cm}}$ so $s=4.85\text{ cm}$. $s'=-(3.25)(4.85\text{ cm})=-15.8\text{ cm}$. The object is 4.85 cm to the left of the lens. The image is 15.8 cm to the left of the lens and is virtual.

EVALUATE: The image is virtual because the object distance is less than the focal length.

34.35. **IDENTIFY:** First use the figure that accompanies the problem to decide if each radius of curvature is positive or negative. Then apply the lensmaker's formula to calculate the focal length of each lens.

SET UP: Use $\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ to calculate f and then use $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ to locate the image. $s=18.0\text{ cm}$.

EXECUTE: **(a)** $\frac{1}{f}=(0.5)\left(\frac{1}{10.0\text{ cm}}-\frac{1}{-15.0\text{ cm}}\right)$ and $f=+12.0\text{ cm}$. $\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}=\frac{s-f}{sf}$.
$s'=\frac{f}{s-f}=\frac{(18.0\text{ cm})(12.0\text{ cm})}{18.0\text{ cm}-12.0\text{ cm}}=+36.0\text{ cm}$. The image is 36.0 cm to the right of the lens.

(b) $\frac{1}{f}=(0.5)\left(\frac{1}{10.0\text{ cm}}-\frac{1}{\infty}\right)$ so $f=+20.0$ cm. $s'=\frac{sf}{s-f}=\frac{(18.0\text{ cm})(20.0\text{ cm})}{18.0\text{ cm}-20.0\text{ cm}}=-180$ cm. The image is 180 cm to the left of the lens.

(c) $\frac{1}{f}=(0.5)\left(\frac{1}{-15.0\text{ cm}}-\frac{1}{10.0\text{ cm}}\right)$ so $f=-12.0$ cm. $s'=\frac{sf}{s-f}=\frac{(18.0\text{ cm})(-12.0\text{ cm})}{18.0\text{ cm}+12.0\text{ cm}}=-7.20$ cm.
The image is 7.20 cm to the left of the lens.

(d) $\frac{1}{f}=(0.5)\left(\frac{1}{-10.0\text{ cm}}-\frac{1}{-15.0\text{ cm}}\right)$ so $f=-60.0$ cm. $s'=\frac{sf}{s-f}=\frac{(18.0\text{ cm})(-60.0\text{ cm})}{18.0\text{ cm}+60.0\text{ cm}}=-13.8$ cm.
The image is 13.8 cm to the left of the lens.

EVALUATE: The focal length of a lens is determined by *both* of its radii of curvature.

34.36. **IDENTIFY:** Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ and $m=\frac{y'}{y}=-\frac{s'}{s}$.

SET UP: $f=+12.0$ cm and $s'=-17.0$ cm.

EXECUTE: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{s}=\frac{1}{12.0\text{ cm}}-\frac{1}{-17.0\text{ cm}}\Rightarrow s=7.0$ cm.

$m=-\frac{s'}{s}=-\frac{(-17.0)}{7.0}=+2.4\Rightarrow y=\frac{y'}{m}=\frac{0.800\text{ cm}}{+2.4}=+0.34$ cm, so the object is 0.34 cm tall, erect, same side as the image. The principal-ray diagram is sketched in Figure 34.36. The image is erect.

EVALUATE: When the object is inside the focal point, a converging lens forms a virtual, enlarged image.

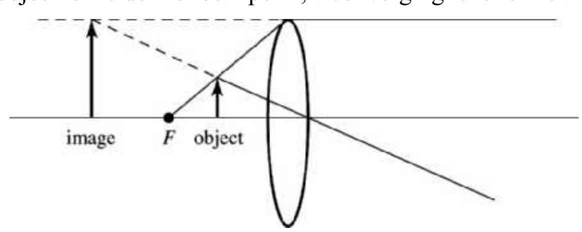


Figure 34.36

34.37. **IDENTIFY:** Use Eq. (34.16) to calculate the object distance s. m calculated from Eq. (34.17) determines the size and orientation of the image.

SET UP: $f=-48.0$ cm. Virtual image 17.0 cm from lens so $s'=-17.0$ cm.

EXECUTE: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$, so $\frac{1}{s}=\frac{1}{f}-\frac{1}{s'}=\frac{s'-f}{s'f}$

$$s=\frac{s'f}{s'-f}=\frac{(-17.0\text{ cm})(-48.0\text{ cm})}{-17.0\text{ cm}-(-48.0\text{ cm})}=+26.3\text{ cm}$$

$$m=-\frac{s'}{s}=-\frac{-17.0\text{ cm}}{+26.3\text{ cm}}=+0.646$$

$$m=\frac{y'}{y}\text{ so }|y|=\frac{|y'|}{|m|}=\frac{8.00\text{ mm}}{0.646}=12.4\text{ mm}$$

The principal-ray diagram is sketched in Figure 34.37.

EVALUATE: Virtual image, real object $(s>0)$ so image and object are on same side of lens.
$m>0$ so image is erect with respect to the object. The height of the object is 12.4 mm.

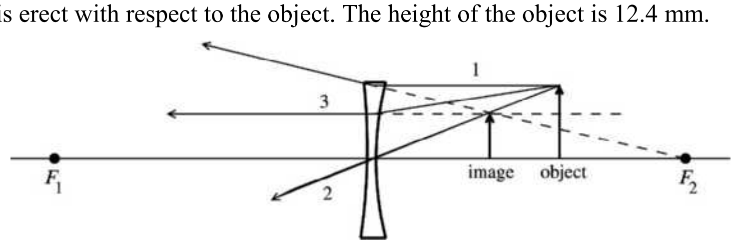


Figure 34.37

34.38. **IDENTIFY:** Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$.

SET UP: The sign of f determines whether the lens is converging or diverging. $s=16.0$ cm. $s'=+36.0$ cm. Use $m=-\frac{s'}{s}$ to find the size and orientation of the image.

EXECUTE: **(a)** $f=\frac{ss'}{s+s'}=\frac{(16.0\text{ cm})(36.0\text{ cm})}{16.0\text{ cm}+36.0\text{ cm}}=11.1\text{ cm}$. $f>0$ and the lens is converging.

(b) $m=-\frac{s'}{s}=-\frac{36.0\text{ cm}}{16.0\text{ cm}}=-2.25$. $|y'|=|m|\,y=(2.25)(8.00\text{ mm})=18.0\text{ mm}$. $m<0$ so the image is inverted.

(c) The principal-ray diagram is sketched in Figure 34.38.

EVALUATE: The image is real so the lens must be converging.

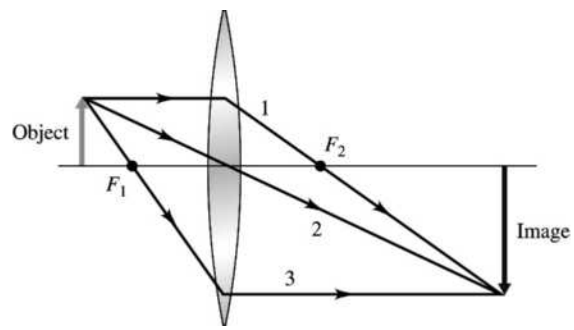


Figure 34.38

34.39. **IDENTIFY:** The first lens forms an image that is then the object for the second lens.

SET UP: Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ to each lens. $m_1=\frac{y_1'}{y_1}$ and $m_2=\frac{y_2'}{y_2}$.

EXECUTE: **(a)** *Lens 1:* $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ gives $s_1'=\frac{s_1 f_1}{s_1-f_1}=\frac{(50.0\text{ cm})(40.0\text{ cm})}{50.0\text{ cm}-40.0\text{ cm}}=+200\text{ cm}$.

$m_1=-\frac{s_1'}{s_1}=-\frac{200\text{ cm}}{50\text{ cm}}=-4.00$. $y_1'=m_1y_1=(-4.00)(1.20\text{ cm})=-4.80\text{ cm}$. The image I_1 is 200 cm to the right of lens 1, is 4.80 cm tall and is inverted.

(b) *Lens 2:* $y_2=-4.80$ cm. The image I_1 is $300\text{ cm}-200\text{ cm}=100\text{ cm}$ to the left of lens 2, so $s_2=+100$ cm. $s_2'=\frac{s_2 f_2}{s_2-f_2}=\frac{(100\text{ cm})(60.0\text{ cm})}{100\text{ cm}-60.0\text{ cm}}=+150\text{ cm}$. $m_2=-\frac{s_2'}{s_2}=-\frac{150\text{ cm}}{100\text{ cm}}=-1.50$.

$y_2'=m_2y_2=(-1.50)(-4.80\text{ cm})=+7.20\text{ cm}$. The image is 150 cm to the right of the second lens, is 7.20 cm tall, and is erect with respect to the original object.

EVALUATE: The overall magnification of the lens combination is $m_{\text{tot}}=m_1m_2$.

34.40. **IDENTIFY:** The first lens forms an image that is then the object for the second lens. We follow the same general procedure as in Problem 34.39.

SET UP: Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ to each lens. $m_1=\frac{y_1'}{y_1}$ and $m_2=\frac{y_2'}{y_2}$. For a diverging lens, $f<0$.

EXECUTE: **(a)** $f_1=+40.0$ cm. I_1 is the same as in Problem 34.39. For lens 2,

$s_2'=\frac{s_2 f_2}{s_2-f_2}=\frac{(100\text{ cm})(-60.0\text{ cm})}{100\text{ cm}-(-60.0\text{ cm})}=-37.5\text{ cm}$. $m_2=-\frac{s_2'}{s_2}=-\frac{-37.5\text{ cm}}{100\text{ cm}}=+0.375$.

$y_2'=m_2y_2=(+0.375)(-4.80\text{ cm})=-1.80\text{ cm}$. The final image is 37.5 cm to the left of the second lens (262.5 cm to the right of the first lens). The final image is inverted and is 1.80 cm tall.

(b) $f_1 = -40.0\text{ cm}$. $s_1' = \dfrac{s_1 f_1}{s_1 - f_1} = \dfrac{(50.0\text{ cm})(-40.0\text{ cm})}{50.0\text{ cm} - (-40.0\text{ cm})} = -22.2\text{ cm}$. $m_1 = -\dfrac{s_1'}{s_1} = -\dfrac{-22.2\text{ cm}}{50.0\text{ cm}} = +0.444$.

$y_1' = m_1 y_1 = (0.444)(1.20\text{ cm}) = 0.533\text{ cm}$. The image I_1 is 22.2 cm to the left of lens 1 so is $22.2\text{ cm} + 300\text{ cm} = 322.2\text{ cm}$ to the left of lens 2 and $s_2 = +322.2\text{ cm}$. $y_2 = y_1' = 0.533\text{ cm}$.

$s_2' = \dfrac{s_2 f_2}{s_2 - f_2} = \dfrac{(322.2\text{ cm})(60.0\text{ cm})}{322.2\text{ cm} - 60.0\text{ cm}} = +73.7\text{ cm}$. $m_2 = -\dfrac{s_2'}{s_2} = -\dfrac{73.7\text{ cm}}{322.2\text{ cm}} = -0.229$.

$y_2' = m_2 y_2 = (-0.229)(0.533\text{ cm}) = -0.122\text{ cm}$. The final image is 73.7 cm to the right of the second lens, is inverted and is 0.122 cm tall.

(c) $f_1 = -40.0\text{ cm}$. $f_2 = -60.0\text{ cm}$. I_1 is as calculated in part (b).

$s_2' = \dfrac{s_2 f_2}{s_2 - f_2} = \dfrac{(322.2\text{ cm})(-60.0\text{ cm})}{322.2\text{ cm} - (-60.0\text{ cm})} = -50.6\text{ cm}$. $m_2 = -\dfrac{s_2'}{s_2} = -\dfrac{-50.6\text{ cm}}{322.2\text{ cm}} = +0.157$.

$y_2' = m_2 y_2 = (0.157)(0.533\text{ cm}) = 0.0837\text{ cm}$. The final image is 50.6 cm to the left of the second lens (249.4 cm to the right of the first lens), is upright and is 0.0837 cm tall.

EVALUATE: The overall magnification of the lens combination is $m_{tot} = m_1 m_2$.

34.41. **IDENTIFY:** The first lens forms an image that is then the object for the second lens. We follow the same general procedure as in Problem 34.39.

SET UP: $m_{tot} = m_1 m_2$. $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ gives $s' = \dfrac{sf}{s - f}$.

EXECUTE: **(a)** *Lens 1:* $f_1 = -12.0\text{ cm}$, $s_1 = 20.0\text{ cm}$. $s' = \dfrac{(20.0\text{ cm})(-12.0\text{ cm})}{20.0\text{ cm} + 12.0\text{ cm}} = -7.5\text{ cm}$.

$m_1 = -\dfrac{s_1'}{s_1} = -\dfrac{-7.5\text{ cm}}{20.0\text{ cm}} = +0.375$.

Lens 2: The image of lens 1 is 7.5 cm to the left of lens 1 so is $7.5\text{ cm} + 9.00\text{ cm} = 16.5\text{ cm}$ to the left of lens 2.

$s_2 = +16.5\text{ cm}$. $f_2 = +12.0\text{ cm}$. $s_2' = \dfrac{(16.5\text{ cm})(12.0\text{ cm})}{16.5\text{ cm} - 12.0\text{ cm}} = 44.0\text{ cm}$. $m_2 = -\dfrac{s_2'}{s_2} = -\dfrac{44.0\text{ cm}}{16.5\text{ cm}} = -2.67$. The final image is 44.0 cm to the right of lens 2 so is 53.0 cm to the right of the first lens.

(b) $s_2' > 0$ so the final image is real.

(c) $m_{tot} = m_1 m_2 = (+0.375)(-2.67) = -1.00$. The image is 2.50 mm tall and is inverted.

EVALUATE: The light travels through the lenses in the direction from left to right. A real image for the second lens is to the right of that lens and a virtual image is to the left of the second lens.

34.42. **IDENTIFY:** The projector lens can be modeled as a thin lens.

SET UP: The thin-lens equation is $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$, and the magnification of the lens is $m = -\dfrac{s'}{s}$.

EXECUTE: **(a)** $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f} \Rightarrow \dfrac{1}{f} = \dfrac{1}{0.150\text{ m}} + \dfrac{1}{9.00\text{ m}} \Rightarrow f = 147.5\text{ mm}$, , so use a $f = 148\text{ mm}$ lens.

(b) $m = -\dfrac{s'}{s} \Rightarrow |m| = 60 \Rightarrow$ Area $= 1.44\text{ m} \times 2.16\text{ m}$.

EVALUATE: The lens must produce a real image to be viewed on the screen. Since the magnification comes out negative, the slides to be viewed must be placed upside down in the tray.

34.43. **IDENTIFY:** Apply $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$.

SET UP: The image is to be formed on the film, so $s' = +20.4\text{ cm}$.

EXECUTE: $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f} \Rightarrow \dfrac{1}{s} + \dfrac{1}{20.4\text{ cm}} = \dfrac{1}{20.0\text{ cm}} \Rightarrow s = 1020\text{ cm} = 10.2\text{ m}$.

EVALUATE: The object distance is much greater than *f*, so the image is just outside the focal point of the lens.

34.44. **IDENTIFY:** Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ and $m=\frac{y'}{y}=-\frac{s'}{s}.$

SET UP: $s=3.90\text{ m}.$ $f=0.085\text{ m}.$

EXECUTE: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{3.90\text{ m}}+\frac{1}{s'}=\frac{1}{0.085\text{ m}}\Rightarrow s'=0.0869\text{ m}.$

$y'=-\frac{s'}{s}y=-\frac{0.0869}{3.90}1750\text{ mm}=-39.0\text{ mm},$ so it will not fit on the $24\text{-mm}\times 36\text{-mm}$ film.

EVALUATE: The image is just outside the focal point and $s'\approx f.$ To have $|y'|=36\text{ mm},$ so that the image will fit on the film, $s=-\frac{s'y}{y'}\approx-\frac{(0.085\text{ m})(1.75\text{ m})}{-0.036\text{ m}}=4.1\text{ m}.$ The person would need to stand about 4.1 m from the lens.

34.45. **IDENTIFY:** $|m|=\left|\frac{s'}{s}\right|.$

SET UP: $s\gg f,$ so $s'\approx f.$

EXECUTE: **(a)** $|m|=\frac{s'}{s}\approx\frac{f}{s}\Rightarrow|m|=\frac{28\text{ mm}}{200{,}000\text{ mm}}=1.4\times10^{-4}.$

(b) $|m|=\frac{s'}{s}\approx\frac{f}{s}\Rightarrow|m|=\frac{105\text{ mm}}{200{,}000\text{ mm}}=5.3\times10^{-4}.$

(c) $|m|=\frac{s'}{s}\approx\frac{f}{s}\Rightarrow|m|=\frac{300\text{ mm}}{200{,}000\text{ mm}}=1.5\times10^{-3}.$

EVALUATE: The magnitude of the magnification increases when *f* increases.

34.46. **IDENTIFY:** $|m|=\left|\frac{s'}{s}\right|=\frac{|y'|}{y}$

SET UP: $s\gg f,$ so $s'\approx f.$

EXECUTE: $|y'|=\frac{s'}{s}y\approx\frac{f}{s}y=\frac{5.00\text{ m}}{9.50\times10^{3}\text{ m}}(70.7\text{ m})=0.0372\text{ m}=37.2\text{ mm}.$

EVALUATE: A very long focal length lens is needed to photograph a distant object.

34.47. **IDENTIFY** and **SET UP:** Find the lateral magnification that results in this desired image size. Use Eq. (34.17) to relate *m* and s' and Eq. (34.16) to relate *s* and s' to *f*.

EXECUTE: **(a)** We need $m=-\frac{24\times10^{-3}\text{ m}}{160\text{ m}}=-1.5\times10^{-4}.$ Alternatively, $m=-\frac{36\times10^{-3}\text{ m}}{240\text{ m}}=-1.5\times10^{-4}.$

$s\gg f$ so $s'\approx f$

Then $m=-\frac{s'}{s}=-\frac{f}{s}=-1.5\times10^{-4}$ and $f=(1.5\times10^{-4})(600\text{ m})=0.090\text{ m}=90\text{ mm}.$

A smaller *f* means a smaller s' and a smaller *m*, so with $f=85\text{ mm}$ the object's image nearly fills the picture area.

(b) We need $m=-\frac{36\times10^{-3}\text{ m}}{9.6\text{ m}}=-3.75\times10^{-3}.$ Then, as in part (a), $\frac{f}{s}=3.75\times10^{-3}$ and

$f=(40.0\text{ m})(3.75\times10^{-3})=0.15\text{ m}=150\text{ mm}.$ Therefore use the 135-mm lens.

EVALUATE: When $s\gg f$ and $s'\approx f,$ $y'=-f(y/s).$ For the mobile home *y*/*s* is smaller so a larger *f* is needed. Note that *m* is very small; the image is much smaller than the object.

34.48. **IDENTIFY:** Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ to each lens. The image of the first lens serves as the object for the second lens.

SET UP: For a distant object, $s\to\infty.$

EXECUTE: **(a)** $s_1=\infty\Rightarrow s_1'=f_1=12\text{ cm}.$

(b) $s_2=4.0\text{ cm}-12\text{ cm}=-8\text{ cm}.$

(c) $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{-8\text{ cm}}+\frac{1}{s_2'}=\frac{1}{-12\text{ cm}} \Rightarrow s_2'=24\text{ cm}$, to the right.

(d) $s_1=\infty \Rightarrow s_1'=f_1=12\text{ cm}$. $s_2=8.0\text{ cm}-12\text{ cm}=-4\text{ cm}$.

$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{-4\text{ cm}}+\frac{1}{s_2'}=\frac{1}{-12\text{ cm}} \Rightarrow s_2'=6\text{ cm}$.

EVALUATE: In each case the image of the first lens serves as a virtual object for the second lens, and $s_2<0$.

34.49. **IDENTIFY:** The *f*-number of a lens is the ratio of its focal length to its diameter. To maintain the same exposure, the amount of light passing through the lens during the exposure must remain the same.

SET UP: The *f*-number is *f/D*.

EXECUTE: **(a)** $f\text{-number}=\frac{f}{D} \Rightarrow f\text{-number}=\frac{180.0\text{ mm}}{16.36\text{ mm}} \Rightarrow f\text{-number}=f/11$. (The *f*-number is an integer.)

(b) *f*/11 to *f*/2.8 is four steps of 2 in intensity, so one needs $1/16^{\text{th}}$ the exposure. The exposure should be $1/480\text{ s}=2.1\times10^{-3}\text{ s}=2.1\text{ ms}$.

EVALUATE: When opening the lens from *f*/11 to *f*/2.8, the area increases by a factor of 16, so 16 times as much light is allowed in. Therefore the exposure time must be decreased by a factor of 1/16 to maintain the same exposure on the film or light receptors of a digital camera.

34.50. **IDENTIFY** and **SET UP:** The square of the aperture diameter is proportional to the length of the exposure time required.

EXECUTE: $\left(\frac{1}{30}\text{ s}\right)\left(\frac{8\text{ mm}}{23.1\text{ mm}}\right)^2 \approx \left(\frac{1}{250}\text{ s}\right)$

EVALUATE: An increase in the aperture diameter decreases the exposure time.

34.51. **IDENTIFY** and **SET UP:** Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ to calculate s'.

EXECUTE: **(a)** A real image is formed at the film, so the lens must be convex.

(b) $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ so $\frac{1}{s'}=\frac{s-f}{sf}$ and $s'=\frac{sf}{s-f}$, with $f=+50.0.0\text{ mm}$. For $s=45\text{ cm}=450\text{ mm}, s'=56\text{ mm}$.

For $s=\infty, s'=f=50\text{ mm}$. The range of distances between the lens and film is 50 mm to 56 mm.

EVALUATE: The lens is closer to the film when photographing more distant objects.

34.52. **IDENTIFY:** $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$

SET UP: $n_a=1.00$, $n_b=1.40$. $s=40.0\text{ cm}$, $s'=2.60\text{ cm}$.

EXECUTE: $\frac{1}{40.0\text{ cm}}+\frac{1.40}{2.60\text{ cm}}=\frac{0.40}{R}$ and $R=0.710\text{ cm}$.

EVALUATE: The cornea presents a convex surface to the object, so $R>0$.

34.53. **(a) IDENTIFY:** The purpose of the corrective lens is to take an object 25 cm from the eye and form a virtual image at the eye's near point. Use Eq. (34.16) to solve for the image distance when the object distance is 25 cm.

SET UP: $\frac{1}{f}=+2.75$ diopters means $f=+\frac{1}{2.75}\text{ m}=+0.3636\text{ m}$ (converging lens)

$f=36.36\text{ cm}; s=25\text{ cm}; s'=?$

EXECUTE: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ so

$$s'=\frac{sf}{s-f}=\frac{(25\text{ cm})(36.36\text{ cm})}{25\text{ cm}-36.36\text{ cm}}=-80.0\text{ cm}$$

The eye's near point is 80.0 cm from the eye.

(b) IDENTIFY: The purpose of the corrective lens is to take an object at infinity and form a virtual image of it at the eye's far point. Use Eq. (34.16) to solve for the image distance when the object is at infinity.

SET UP: $\frac{1}{f} = -1.30$ diopters means $f = -\frac{1}{1.30}\text{ m} = -0.7692\text{ m}$ (diverging lens)

$f = -76.92\text{ cm}$; $s = \infty$; $s' = ?$

EXECUTE: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ and $s = \infty$ says $\frac{1}{s'} = \frac{1}{f}$ and $s' = f = -76.9\text{ cm}$. The eye's far point is 76.9 cm from the eye.

EVALUATE: In each case a virtual image is formed by the lens. The eye views this virtual image instead of the object. The object is at a distance where the eye can't focus on it, but the virtual image is at a distance where the eye can focus.

34.54. IDENTIFY and **SET UP:** For an object 25.0 cm from the eye, the corrective lens forms a virtual image at the near point of the eye. $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$. $P(\text{in diopters}) = 1/f$ (in m).

EXECUTE: **(a)** The person is farsighted.

(b) A converging lens is needed.

(c) $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$. $f = \frac{ss'}{s+s'} = \frac{(25.0\text{ cm})(-45.0\text{ cm})}{25.0\text{ cm} - 45.0\text{ cm}} = +56.2\text{ cm}$. The power is $\frac{1}{0.562\text{ m}} = +1.78$ diopters.

EVALUATE: The object is inside the focal point of the lens, so it forms a virtual image.

34.55. IDENTIFY and **SET UP:** For an object 25.0 cm from the eye, the corrective lens forms a virtual image at the near point of the eye. The distances from the corrective lens are $s = 23.0\text{ cm}$ and $s' = -43.0\text{ cm}$. $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$. $P(\text{in diopters}) = 1/f$ (in m).

EXECUTE: Solving $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ for f gives $f = \frac{ss'}{s+s'} = \frac{(23.0\text{ cm})(-43.0\text{ cm})}{23.0\text{ cm} - 43.0\text{ cm}} = +49.4\text{ cm}$. The power is $\frac{1}{0.494\text{ m}} = 2.02$ diopters.

EVALUATE: In Problem 34.54 the contact lenses have power 1.78 diopters. The power of the lenses is different for ordinary glasses versus contact lenses.

34.56. IDENTIFY and **SET UP:** For an object very far from the eye, the corrective lens forms a virtual image at the far point of the eye. $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$. $P(\text{in diopters}) = 1/f$ (in m).

EXECUTE: **(a)** The person is nearsighted.

(b) A diverging lens is needed.

(c) In $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$, $s \to \infty$, so $f = s' = -75.0\text{ cm}$. The power is $\frac{1}{-0.750\text{ m}} = -1.33$ diopters.

EVALUATE: A diverging lens is needed to form a virtual image of a distant object. A converging lens could not do this since distant objects cannot be inside its focal point.

34.57. IDENTIFY and **SET UP:** For an object very far from the eye, the corrective lens forms a virtual image at the far point of the eye. The distances from the lens are $s \to \infty$ and $s' = -73.0\text{ cm}$. $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$. $P(\text{in diopters}) = 1/f$ (in m).

EXECUTE: In $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$, $s \to \infty$, so $f = s' = -73.0\text{ cm}$. The power is $\frac{1}{-0.730\text{ m}} = -1.37$ diopters.

EVALUATE: A diverging lens is needed to form a virtual image of a distant object. A converging lens could not do this since distant objects cannot be inside its focal point.

34.58. IDENTIFY: When the object is at the focal point, $M = \frac{25.0\text{ cm}}{f}$. In part (b), apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ to calculate s for $s' = -25.0\text{ cm}$.

SET UP: Our calculation assumes the near point is 25.0 cm from the eye.

EXECUTE: **(a)** Angular magnification $M = \dfrac{25.0\text{ cm}}{f} = \dfrac{25.0\text{ cm}}{6.00\text{ cm}} = 4.17.$

(b) $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f} \Rightarrow \dfrac{1}{s} + \dfrac{1}{-25.0\text{ cm}} = \dfrac{1}{6.00\text{ cm}} \Rightarrow s = 4.84\text{ cm}.$

EVALUATE: In part (b), $\theta' = \dfrac{y}{s}$, $\theta = \dfrac{y}{25.0\text{ cm}}$ and $M = \dfrac{25.0\text{ cm}}{s} = \dfrac{25.0\text{ cm}}{4.84\text{ cm}} = 5.17.$ M is greater when the image is at the near point than when the image is at infinity.

34.59. **IDENTIFY:** Use Eqs. (34.16) and (34.17) to calculate s and y'.

(a) SET UP: $f = 8.00$ cm; $s' = -25.0$ cm; $s = ?$

$\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$, so $\dfrac{1}{s} = \dfrac{1}{f} - \dfrac{1}{s'} = \dfrac{s' - f}{s'f}$

EXECUTE: $s = \dfrac{s'f}{s' - f} = \dfrac{(-25.0\text{ cm})(+8.00\text{ cm})}{-25.0\text{ cm} - 8.00\text{ cm}} = +6.06\text{ cm}$

(b) $m = -\dfrac{s'}{s} = -\dfrac{-25.0\text{ cm}}{6.06\text{ cm}} = +4.125$

$|m| = \dfrac{|y'|}{|y|}$ so $|y'| = |m||y| = (4.125)(1.00\text{ mm}) = 4.12\text{ mm}$

EVALUATE: The lens allows the object to be much closer to the eye than the near point. The lens allows the eye to view an image at the near point rather than the object.

34.60. **IDENTIFY:** For a thin lens, $-\dfrac{s'}{s} = \dfrac{y'}{y}$, so $\left|\dfrac{y'}{s'}\right| = \left|\dfrac{y}{s}\right|$, and the angular size of the image equals the angular size of the object.

SET UP: The object has angular size $\theta = \dfrac{y}{f}$, with θ in radians.

EXECUTE: $\theta = \dfrac{y}{f} \Rightarrow f = \dfrac{y}{\theta} = \dfrac{2.00\text{ mm}}{0.025\text{ rad}} = 80.0\text{ mm} = 8.00\text{ cm}.$

EVALUATE: If the insect is at the near point of a normal eye, its angular size is $\dfrac{2.00\text{ mm}}{250\text{ mm}} = 0.0080\text{ rad}.$

34.61. **IDENTIFY:** Eq. (34.24) can be written $M = |m_1|M_2 = \left|\dfrac{s_1'}{f_1}\right|M_2.$

SET UP: $s_1' = f_1 + 120\text{ mm}$

EXECUTE: $f = 16\text{ mm}: s' = 120\text{ mm} + 16\text{ mm} = 136\text{ mm}; s = 16\text{ mm}.$ $|m_1| = \dfrac{s'}{s} = \dfrac{136\text{ mm}}{16\text{ mm}} = 8.5.$

$f = 4\text{ mm}: s' = 120\text{ mm} + 4\text{ mm} = 124\text{ mm}; s = 4\text{ mm} \Rightarrow |m_1| = \dfrac{s'}{s} = \dfrac{124\text{ mm}}{4\text{ mm}} = 31.$

$f = 1.9\text{ mm}: s' = 120\text{ mm} + 1.9\text{ mm} = 122\text{ mm}; s = 1.9\text{ mm} \Rightarrow |m_1| = \dfrac{s'}{s} = \dfrac{122\text{ mm}}{1.9\text{ mm}} = 64.$

The eyepiece magnifies by either 5 or 10, so:

(a) The maximum magnification occurs for the 1.9-mm objective and $10\times$ eyepiece:

$M = |m_1|M_e = (64)(10) = 640.$

(b) The minimum magnification occurs for the 16-mm objective and $5\times$ eyepiece:

$M = |m_1|M_e = (8.5)(5) = 43.$

EVALUATE: The smaller the focal length of the objective, the greater the overall magnification.

34.62. **IDENTIFY:** Apply Eq. (34.24).

SET UP: $s_1' = 160\text{ mm} + 5.0\text{ mm} = 165\text{ mm}$

EXECUTE: **(a)** $M = \dfrac{(250\text{ mm})s_1'}{f_1 f_2} = \dfrac{(250\text{ mm})(165\text{ mm})}{(5.00\text{ mm})(26.0\text{ mm})} = 317.$

(b) The minimum separation is $\dfrac{0.10\text{ mm}}{M} = \dfrac{0.10\text{ mm}}{317} = 3.15\times10^{-4}$ mm.

EVALUATE: The angular size of the image viewed by the eye when looking through the microscope is 317 times larger than if the object is viewed at the near-point of the unaided eye.

34.63. **(a) IDENTIFY** and **SET UP:**

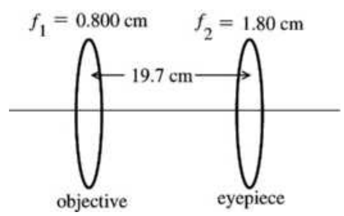


Figure 34.63

Final image is at ∞ so the object for the eyepiece is at its focal point. But the object for the eyepiece is the image of the objective so the image formed by the objective is $19.7\text{ cm} - 1.80\text{ cm} = 17.9\text{ cm}$ to the right of the lens. Apply Eq. (34.16) to the image formation by the objective, solve for the object distance s.

$f = 0.800\text{ cm};\ s' = 17.9\text{ cm};\ s = ?$

$\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$, so $\dfrac{1}{s} = \dfrac{1}{f} - \dfrac{1}{s'} = \dfrac{s' - f}{s'f}$

EXECUTE: $s = \dfrac{s'f}{s' - f} = \dfrac{(17.9\text{ cm})(+0.800\text{ cm})}{17.9\text{ cm} - 0.800\text{ cm}} = +8.37\text{ mm}$

(b) SET UP: Use Eq. (34.17).

EXECUTE: $m_1 = -\dfrac{s'}{s} = -\dfrac{17.9\text{ cm}}{0.837\text{ cm}} = -21.4$

The linear magnification of the objective is 21.4.

(c) SET UP: Use Eq. (34.24): $M = m_1 M_2$

EXECUTE: $M_2 = \dfrac{25\text{ cm}}{f_2} = \dfrac{25\text{ cm}}{1.80\text{ cm}} = 13.9$

$M = m_1 M_2 = (-21.4)(13.9) = -297$

EVALUATE: M is not accurately given by $(25\text{ cm})s_1'/f_1 f_2 = 311$, because the object is not quite at the focal point of the objective ($s_1 = 0.837$ cm and $f_1 = 0.800$ cm).

34.64. **IDENTIFY:** For a telescope, $M = -\dfrac{f_1}{f_2}$.

SET UP: $f_2 = 9.0$ cm. The distance between the two lenses equals $f_1 + f_2$.

EXECUTE: $f_1 + f_2 = 1.80\text{ m} \Rightarrow f_1 = 1.80\text{ m} - 0.0900\text{ m} = 1.71\text{ m}$. $M = -\dfrac{f_1}{f_2} = -\dfrac{171}{9.00} = -19.0.$

EVALUATE: For a telescope, $f_1 \gg f_2$.

34.65. **(a) IDENTIFY** and **SET UP:** Use Eq. (34.25), with $f_1 = 95.0$ cm (objective) and $f_2 = 15.0$ cm (eyepiece).

EXECUTE: $M = -\dfrac{f_1}{f_2} = -\dfrac{95.0\text{ cm}}{15.0\text{ cm}} = -6.33$

(b) IDENTIFY: Use Eq. (34.17) to calculate y'.

SET UP: $s = 3.00\times10^3$ m

$s' = f_1 = 95.0$ cm (since s is very large, $s' \approx f$)

EXECUTE: $m = -\dfrac{s'}{s} = -\dfrac{0.950\text{ m}}{3.00\times 10^3\text{ m}} = -3.167\times 10^{-4}$

$|y'| = |m||y| = (3.167\times 10^{-4})(60.0\text{ m}) = 0.0190\text{ m} = 1.90\text{ cm}$

(c) IDENTIFY: Use Eq. (34.21) and the angular magnification M obtained in part (a) to calculate θ'. The angular size θ of the image formed by the objective (object for the eyepiece) is its height divided by its distance from the objective.

EXECUTE: The angular size of the object for the eyepiece is $\theta = \dfrac{0.0190\text{ m}}{0.950\text{ m}} = 0.0200\text{ rad}.$

(Note that this is also the angular size of the object for the objective: $\theta = \dfrac{60.0\text{ m}}{3.00\times 10^3\text{ m}} = 0.0200\text{ rad}.$ For a thin lens the object and image have the same angular size and the image of the objective is the object for the eyepiece.) $M = \dfrac{\theta'}{\theta}$ (Eq. 34.21) so the angular size of the image is $\theta' = M\theta = -(6.33)(0.0200\text{ rad}) =$ -0.127 rad. (The minus sign shows that the final image is inverted.)

EVALUATE: The lateral magnification of the objective is small; the image it forms is much smaller than the object. But the total angular magnification is larger than 1.00; the angular size of the final image viewed by the eye is 6.33 times larger than the angular size of the original object, as viewed by the unaided eye.

34.66. **IDENTIFY:** The angle subtended by Saturn with the naked eye is the same as the angle subtended by the image of Saturn formed by the objective lens (see Figure 34.53 in the textbook).

SET UP: The angle subtended by Saturn is $\theta = \dfrac{\text{diameter of Saturn}}{\text{distance to Saturn}} = \dfrac{y'}{f_1}.$

EXECUTE: Putting in the numbers gives $\theta = \dfrac{y'}{f_1} = \dfrac{1.7\text{ mm}}{18\text{m}} = \dfrac{0.0017\text{ m}}{18\text{ m}} = 9.4\times 10^{-5}\text{ rad} = 0.0054°.$

EVALUATE: The angle subtended by the final image, formed by the eyepiece, would be much larger than 0.0054°.

34.67. **IDENTIFY:** $f = R/2$ and $M = -\dfrac{f_1}{f_2}.$

SET UP: For object and image both at infinity, $f_1 + f_2$ equals the distance d between the eyepiece and the mirror vertex. $f_2 = 1.10$ cm. $R_1 = 1.30$ m.

EXECUTE: **(a)** $f_1 = \dfrac{R_1}{2} = 0.650\text{ m} \Rightarrow d = f_1 + f_2 = 0.661\text{ m}.$

(b) $|M| = \dfrac{f_1}{f_2} = \dfrac{0.650\text{ m}}{0.011\text{ m}} = 59.1.$

EVALUATE: For a telescope, $f_1 \gg f_2$.

34.68. **IDENTIFY:** Combine $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{2}{R}$ and $m = -\dfrac{s'}{s}.$

SET UP: $m = +2.50.$ $R > 0.$

EXECUTE: $m = -\dfrac{s'}{s} = +2.50.$ $s' = -2.50s.$ $\dfrac{1}{s} + \dfrac{1}{-2.50s} = \dfrac{2}{R}.$ $\dfrac{0.600}{s} = \dfrac{2}{R}$ and $s = 0.300R.$

$s' = -2.50s = (-2.50)(0.300R) = -0.750R.$ The object is a distance of $0.300R$ in front of the mirror and the image is a distance of $0.750R$ behind the mirror.

EVALUATE: For a single mirror an erect image is always virtual.

34.69. **IDENTIFY** and **SET UP:** For a plane mirror $s' = -s.$ $v = \dfrac{ds}{dt}$ and $v' = \dfrac{ds'}{dt},$ so $v' = -v.$

EXECUTE: The velocities of the object and image relative to the mirror are equal in magnitude and opposite in direction. Thus both you and your image are receding from the mirror surface at 3.60 m/s, in opposite directions. Your image is therefore moving at 7.20 m/s relative to you.

EVALUATE: The result derives from the fact that for a plane mirror the image is the same distance behind the mirror as the object is in front of the mirror.

34.70. **IDENTIFY:** Apply the law of reflection.
SET UP: The image of one mirror can serve as the object for the other mirror.
EXECUTE: **(a)** There are three images formed, as shown in Figure 34.70a.
(b) The paths of rays for each image are sketched in Figure 34.70b.
EVALUATE: Our results agree with Figure 34.9 in the textbook.

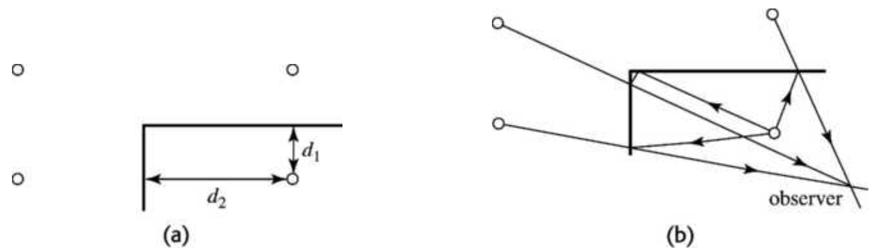


Figure 34.70

34.71. **IDENTIFY:** Apply the law of reflection for rays from the feet to the eyes and from the top of the head to the eyes.
SET UP: In Figure 34.71, ray 1 travels from the feet of the woman to her eyes and ray 2 travels from the top of her head to her eyes. The total height of the woman is h.
EXECUTE: The two angles labeled θ_1 are equal because of the law of reflection, as are the two angles labeled θ_2. Since these angles are equal, the two distances labeled y_1 are equal and the two distances labeled y_2 are equal. The height of the woman is $h_w = 2y_1 + 2y_2$. As the drawing shows, the height of the mirror is $h_m = y_1 + y_2$. Comparing, we find that $h_m = h_w/2$. The minimum height required is half the height of the woman.
EVALUATE: The height of the image is the same as the height of the woman, so the height of the image is twice the height of the mirror.

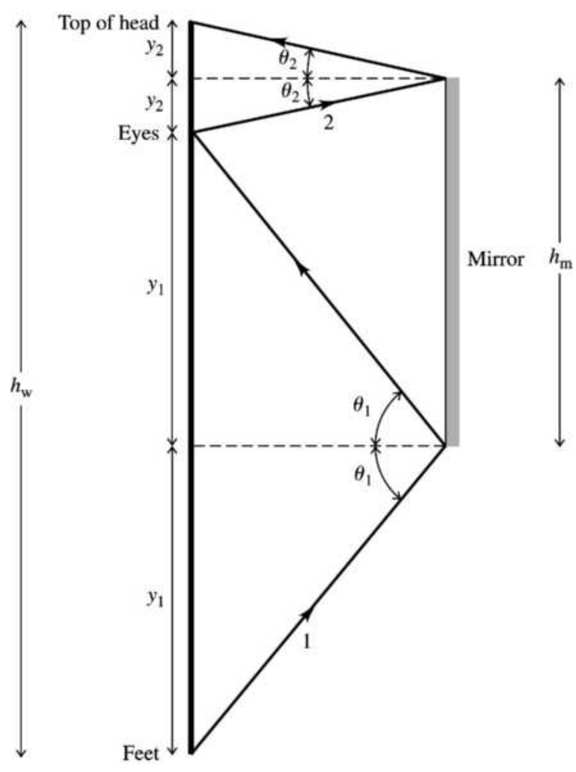


Figure 34.71

34.72. **IDENTIFY:** Apply $\frac{1}{s}+\frac{1}{s'}=\frac{2}{R}$ and $m=-\frac{s'}{s}$.

SET UP: Since the image is projected onto the wall it is real and $s'>0$. $m=-\frac{s'}{s}$ so m is negative and $m=-2.25$. The object, mirror and wall are sketched in Figure 34.72. The sketch shows that $s'-s=300$ cm.

EXECUTE: $m=-2.25=-\frac{s'}{s}$ and $s'=2.25s$. $s'-s=2.25s-s=300$ cm so $s=240$ cm.

$s'=300\text{ cm}+240\text{ cm}=540$ cm. The mirror should be 5.40 m from the wall. $\frac{1}{s}+\frac{1}{s'}=\frac{2}{R}$.

$\frac{1}{240\text{ cm}}+\frac{1}{540\text{ cm}}=\frac{2}{R}$. $R=3.32$ m.

EVALUATE: The focal length of the mirror is $f=R/2=166$ cm and $s>f$, as it must if the image is to be real.

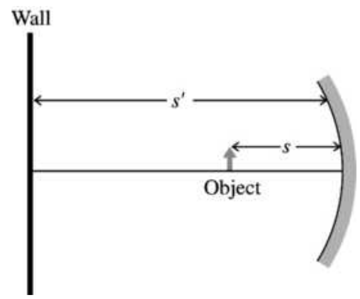


Figure 34.72

34.73. **IDENTIFY:** We are given the image distance, the image height, and the object height. Use Eq. (34.7) to calculate the object distance s. Then use Eq. (34.4) to calculate R.

SET UP: The image is to be formed on screen so it is a real image; $s'>0$. The mirror-to-screen distance is 8.00 m, so $s'=+800$ cm. $m=-\frac{s'}{s}<0$ since both s and s' are positive.

EXECUTE: **(a)** $|m|=\frac{|y'|}{|y|}=\frac{24.0\text{ cm}}{0.600\text{ cm}}=40.0$, so $m=-40.0$. Then $m=-\frac{s'}{s}$ gives

$s=-\frac{s'}{m}=-\frac{800\text{ cm}}{-40.0}=+20.0$ cm.

(b) $\frac{1}{s}+\frac{1}{s'}=\frac{2}{R}$, so $\frac{2}{R}=\frac{s+s'}{ss'}$. $R=2\left(\frac{ss'}{s+s'}\right)=2\left(\frac{(20.0\text{ cm})(800\text{ cm})}{20.0\text{ cm }+800\text{ cm}}\right)=39.0$ cm.

EVALUATE: R is calculated to be positive, which is correct for a concave mirror. Also, in part (a) s is calculated to be positive, as it should be for a real object.

34.74. **IDENTIFY:** Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ to calculate s' and then use $m=-\frac{s'}{s}=\frac{y'}{y}$ to find the height of the image.

SET UP: For a convex mirror, $R<0$, so $R=-18.0$ cm and $f=\frac{R}{2}=-9.00$ cm.

EXECUTE: **(a)** $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$. $s'=\frac{sf}{s-f}=\frac{(900\text{ cm})(-9.00\text{ cm})}{900\text{ cm}-(-9.00\text{ cm})}=-8.91$ cm.

$m=-\frac{s'}{s}=-\frac{-8.91\text{ cm}}{900\text{ cm}}=9.90\times10^{-3}$. $|y'|=|m|y=(9.90\times10^{-3})(1.5\text{ m})=0.0149\text{ m}=1.49$ cm.

(b) The height of the image is much less than the height of the car, so the car appears to be farther away than its actual distance.

EVALUATE: A plane mirror would form an image the same size as the car. Since the image formed by the convex mirror is smaller than the car, the car appears to be farther away compared to what it would appear using a plane mirror.

34.75. **IDENTIFY:** Apply $\frac{1}{s}+\frac{1}{s'}=\frac{2}{R}$ and $m=-\frac{s'}{s}$.

SET UP: $R=+19.4$ cm.

EXECUTE: **(a)** $\frac{1}{s}+\frac{1}{s'}=\frac{2}{R}\Rightarrow\frac{1}{8.0\text{ cm}}+\frac{1}{s'}=\frac{2}{19.4\text{ cm}}\Rightarrow s'=-46\text{ cm},$ so the image is virtual.

(b) $m=-\frac{s'}{s}=-\frac{-46}{8.0}=5.8,$ so the image is erect, and its height is $y'=(5.8)y=(5.8)(5.0\text{ mm})=29\text{ mm}.$

EVALUATE: **(c)** When the filament is 8 cm from the mirror, the image is virtual and cannot be projected onto a wall.

34.76. **IDENTIFY:** Apply $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$, with $R\to\infty$ since the surfaces are flat.

SET UP: The image formed by the first interface serves as the object for the second interface.

EXECUTE: For the water-benzene interface, we get the apparent water depth:
$\frac{n_a}{s}+\frac{n_b}{s'}=0\Rightarrow\frac{1.33}{6.50\text{ cm}}+\frac{1.50}{s'}=0\Rightarrow s'=-7.33\text{ cm}.$ For the benzene-air interface, we get the total apparent distance to the bottom: $\frac{n_a}{s}+\frac{n_b}{s'}=0\Rightarrow\frac{1.50}{(7.33\text{ cm}+4.20\text{ cm})}+\frac{1}{s'}=0\Rightarrow s'=-7.69\text{ cm}.$

EVALUATE: At the water-benzene interface the light refracts into material of greater refractive index and the overall effect is that the apparent depth is greater than the actual depth.

34.77. **IDENTIFY:** Since the truck is moving toward the mirror, its image will also be moving toward the mirror.
SET UP: The equation relating the object and image distances to the focal length of a spherical mirror is $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$, where $f=R/2$.

EXECUTE: Since the mirror is convex, $f=R/2=(-1.50\text{ m})/2=-0.75\text{ m}.$ Applying the equation for a spherical mirror gives $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow s'=\frac{fs}{s-f}.$ Using the chain rule from calculus and the fact that $v=ds/dt,$ we have $v'=\frac{ds'}{dt}=\frac{ds'}{ds}\frac{ds}{dt}=v\frac{f^2}{(s-f)^2}.$ Solving for v gives

$v=v'\left(\frac{s-f}{f}\right)^2=(1.9\text{ m/s})\left[\frac{2.0\text{ m}-(-0.75\text{ m})}{-0.75\text{ m}}\right]^2=25.5\text{ m/s}.$ This is the velocity of the truck relative to the mirror, so the truck is approaching the mirror at 25.5 m/s. You are traveling at 25 m/s, so the truck must be traveling at $25\text{ m/s}+25.5\text{ m/s}=51\text{ m/s}$ relative to the highway.

EVALUATE: Even though the truck and car are moving at constant speed, the image of the truck is *not* moving at constant speed because its location depends on the distance from the mirror to the truck.

34.78. **IDENTIFY:** Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ and the concept of principal rays.

SET UP: $s=10.0$ cm. If extended backwards the ray comes from a point on the optic axis 18.0 cm from the lens and the ray is parallel to the optic axis after it passes through the lens.

EXECUTE: **(a)** The ray is bent toward the optic axis by the lens so the lens is converging.

(b) The ray is parallel to the optic axis after it passes through the lens so it comes from the focal point; $f=18.0$ cm.

(c) The principal-ray diagram is drawn in Figure 34.78. The diagram shows that the image is 22.5 cm to the left of the lens.

(d) $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ gives $s'=\frac{sf}{s-f}=\frac{(10.0\text{ cm})(18.0\text{ cm})}{10.0\text{ cm}-18.0\text{ cm}}=-22.5\text{ cm}.$ The calculated image position agrees with the principal ray diagram.

EVALUATE: The image is virtual. A converging lens produces a virtual image when the object is inside the focal point.

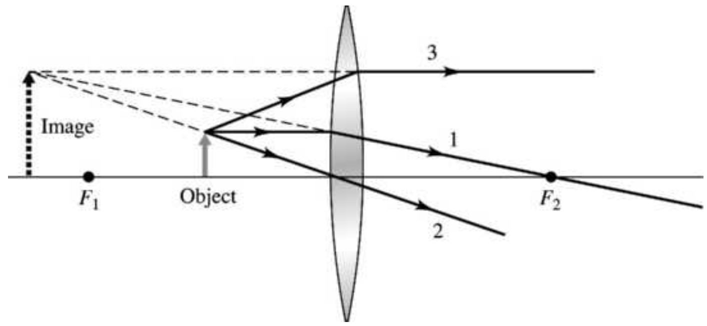


Figure 34.78

34.79. **IDENTIFY** and **SET UP:** Rays that pass through the hole are undeflected. All other rays are blocked. $m = -\frac{s'}{s}.$

EXECUTE: **(a)** The ray diagram is drawn in Figure 34.79. The ray shown is the only ray from the top of the object that reaches the film, so this ray passes through the top of the image. An inverted image is formed on the far side of the box, no matter how far this side is from the pinhole and no matter how far the object is from the pinhole.

(b) $s = 1.5$ m. $s' = 20.0$ cm. $m = -\frac{s'}{s} = -\frac{20.0\text{ cm}}{150\text{ cm}} = -0.133$. $y' = my = (-0.133)(18\text{ cm}) = -2.4$ cm.

The image is 2.4 cm tall.

EVALUATE: A defect of this camera is that not much light energy passes through the small hole each second, so long exposure times are required.

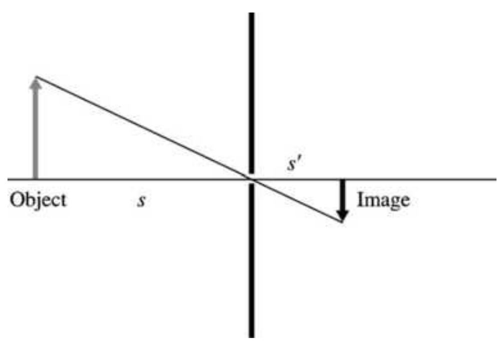


Figure 34.79

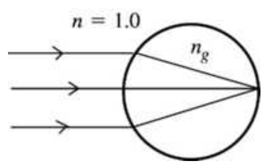
34.80. **IDENTIFY:** In this context, the microscope just looks at an image or object. Apply $\frac{n_a}{s} + \frac{n_b}{s'} = 0$ to the image formed by refraction at the top surface of the second plate. In this calculation the object is the bottom surface of the second plate.

SET UP: The thickness of the second plate is $2.50\text{ mm} + 0.78$ mm, and this is s. The image is 2.50 mm below the top surface, so $s' = -2.50$ mm.

EXECUTE: $\frac{n_a}{s} + \frac{n_b}{s'} = 0 \Rightarrow \frac{n}{s} + \frac{1}{s'} = 0 \Rightarrow n = -\frac{s}{s'} = -\frac{2.50\text{ mm} + 0.780\text{ mm}}{-2.50\text{ mm}} = 1.31.$

EVALUATE: The object and image distances are measured from the front surface of the second plate, and the image is virtual.

34.81. **IDENTIFY:** Apply Eq. (34.11) to the image formed by refraction at the front surface of the sphere.
SET UP: Let n_g be the index of refraction of the glass. The image formation is shown in Figure 34.81.



$s = \infty$
$s' = +2r$, where r is the radius of the sphere
$n_a = 1.00, n_b = n_g, R = +r$

Figure 34.81

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$

EXECUTE: $\frac{1}{\infty} + \frac{n_g}{2r} = \frac{n_g - 1.00}{r}$

$\frac{n_g}{2r} = \frac{n_g}{r} - \frac{1}{r}; \frac{n_g}{2r} = \frac{1}{r}$ and $n_g = 2.00$

EVALUATE: The required refractive index of the glass does not depend on the radius of the sphere.

34.82. **IDENTIFY:** Apply $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$ and $m = -\frac{n_a s'}{n_b s}$ to each refraction. The overall magnification is $m = m_1 m_2$.

SET UP: For the first refraction, $R = +6.0$ cm, $n_a = 1.00$ and $n_b = 1.60$. For the second refraction, $R = -12.0$ cm, $n_a = 1.60$ and $n_b = 1.00$.

EXECUTE: **(a)** The image from the left end acts as the object for the right end of the rod.

(b) $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \Rightarrow \frac{1}{23.0\text{ cm}} + \frac{1.60}{s'} = \frac{0.60}{6.0\text{ cm}} \Rightarrow s' = 28.3$ cm.

So the second object distance is $s_2 = 40.0\text{ cm} - 28.3\text{ cm} = 11.7$ cm. $m_1 = -\frac{n_a s'}{n_b s} = -\frac{28.3}{(1.60)(23.0)} = -0.769.$

(c) The object is real and inverted.

(d) $\frac{n_a}{s_2} + \frac{n_b}{s_2'} = \frac{n_b - n_a}{R} \Rightarrow \frac{1.60}{11.7\text{ cm}} + \frac{1}{s_2'} = \frac{-0.60}{-12.0\text{ cm}} \Rightarrow s' = -11.5$ cm.

$m_2 = -\frac{n_a s'}{n_b s} = -\frac{(1.60)(-11.5)}{11.7} = 1.57 \Rightarrow m = m_1 m_2 = (-0.769)(1.57) = -1.21.$

(e) The final image is virtual, and inverted.
(f) $y' = (1.50\text{ mm})(-1.21) = -1.82$ mm.

EVALUATE: The first image is to the left of the second surface, so it serves as a real object for the second surface, with positive object distance.

34.83. **IDENTIFY:** Apply Eqs. (34.11) and (34.12) to the refraction as the light enters the rod and as it leaves the rod. The image formed by the first surface serves as the object for the second surface. The total magnification is $m_{tot} = m_1 m_2$, where m_1 and m_2 are the magnifications for each surface.

SET UP: The object and rod are shown in Figure 34.83.

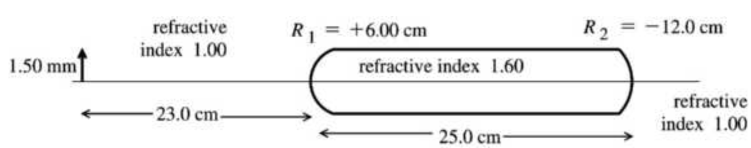


Figure 34.83

(a) image formed by refraction at first surface (left end of rod):
$s = +23.0$ cm; $n_a = 1.00$; $n_b = 1.60$; $R = +6.00$ cm

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$

EXECUTE: $\frac{1}{23.0\text{ cm}} + \frac{1.60}{s'} = \frac{1.60 - 1.00}{6.00\text{ cm}}$

$$\frac{1.60}{s'} = \frac{1}{10.0\text{ cm}} - \frac{1}{23.0\text{ cm}} = \frac{23 - 10}{230\text{ cm}} = \frac{13}{230\text{ cm}}$$

$s' = 1.60\left(\frac{230\text{ cm}}{13}\right) = +28.3$ cm; image is 28.3 cm to right of first vertex.

This image serves as the object for the refraction at the second surface (right-hand end of rod). It is $28.3\text{ cm} - 25.0\text{ cm} = 3.3\text{ cm}$ to the right of the second vertex. For the second surface $s = -3.3$ cm (virtual object).
(b) EVALUATE: Object is on side of outgoing light, so is a virtual object.
(c) SET UP: Image formed by refraction at second surface (right end of rod):
$s = -3.3$ cm; $n_a = 1.60$; $n_b = 1.00$; $R = -12.0$ cm

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$

EXECUTE: $\frac{1.60}{-3.3\text{ cm}} + \frac{1.00}{s'} = \frac{1.00 - 1.60}{-12.0\text{ cm}}$

$s' = +1.9$ cm; $s' > 0$ so image is 1.9 cm to right of vertex at right-hand end of rod.
(d) $s' > 0$ so final image is real.
Magnification for first surface:

$$m_1 = -\frac{n_a s'}{n_b s} = -\frac{(1.00)(+28.3\text{ cm})}{(1.60)(+23.0\text{ cm})} = -0.769$$

Magnification for second surface:

$$m_2 = -\frac{n_a s'}{n_b s} = -\frac{(1.60)(+1.9\text{ cm})}{(1.00)(-3.3\text{ cm})} = +0.92$$

The overall magnification is $m_{tot} = m_1 m_2 = (-0.769)(+0.92) = -0.71$ $m_{tot} < 0$ so final image is inverted with respect to the original object.
(e) $y' = m_{tot} y = (-0.71)(1.50\text{ mm}) = -1.06$ mm
The final image has a height of 1.06 mm.
EVALUATE: The two refracting surfaces are not close together and Eq. (34.18) does not apply.

34.84. **IDENTIFY:** Apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ and $m = \frac{y'}{y} = -\frac{s'}{s}$. The type of lens determines the sign of *f*. The sign of s' determines whether the image is real or virtual.
SET UP: $s = +8.00$ cm. $s' = -3.00$ cm. s' is negative because the image is on the same side of the lens as the object.
EXECUTE: **(a)** $\frac{1}{f} = \frac{s + s'}{ss'}$ and $f = \frac{ss'}{s + s'} = \frac{(8.00\text{ cm})(-3.00\text{ cm})}{8.00\text{ cm} - 3.00\text{ cm}} = -4.80$ cm. *f* is negative so the lens is diverging.
(b) $m = -\frac{s'}{s} = -\frac{-3.00\text{ cm}}{8.00\text{ cm}} = +0.375$. $y' = my = (0.375)(6.50\text{ mm}) = 2.44$ mm. $s' < 0$ and the image is virtual.
EVALUATE: A converging lens can also form a virtual image, if the object distance is less than the focal length. But in that case $|s'| > s$ and the image would be farther from the lens than the object is.

34.85. **IDENTIFY:** $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$. The type of lens determines the sign of *f*. $m = \frac{y'}{y} = -\frac{s'}{s}$. The sign of s' depends on whether the image is real or virtual. $s = 16.0$ cm.
SET UP: $s' = -22.0$ cm; s' is negative because the image is on the same side of the lens as the object.

EXECUTE: **(a)** $\frac{1}{f} = \frac{s+s'}{ss'}$ and $f = \frac{ss'}{s+s'} = \frac{(16.0\text{ cm})(-22.0\text{ cm})}{16.0\text{ cm} - 22.0\text{ cm}} = +58.7\text{ cm}$. f is positive so the lens is converging.

(b) $m = -\frac{s'}{s} = -\frac{-22.0\text{ cm}}{16.0\text{ cm}} = 1.38$. $y' = my = (1.38)(3.25\text{ mm}) = 4.48\text{ mm}$. $s' < 0$ and the image is virtual.

EVALUATE: A converging lens forms a virtual image when the object is closer to the lens than the focal point.

34.86. **IDENTIFY:** Apply $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$. Use the image distance when viewed from the flat end to determine the refractive index n of the rod.

SET UP: When viewing from the flat end, $n_a = n$, $n_b = 1.00$ and $R \to \infty$. When viewing from the curved end, $n_a = n$, $n_b = 1.00$ and $R = -10.0\text{ cm}$.

EXECUTE: $\frac{n_a}{s} + \frac{n_b}{s'} = 0 \Rightarrow \frac{n}{15.0\text{ cm}} + \frac{1}{-9.50\text{ cm}} = 0 \Rightarrow n = \frac{15.0}{9.50} = 1.58$. When viewed from the curved end of the rod $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \Rightarrow \frac{n}{s} + \frac{1}{s'} = \frac{1-n}{R} \Rightarrow \frac{1.58}{15.0\text{ cm}} + \frac{1}{s'} = \frac{-0.58}{-10.0\text{ cm}}$, and $s' = -21.1\text{ cm}$. The image is 21.1 cm within the rod from the curved end.

EVALUATE: In each case the image is virtual and on the same side of the surface as the object.

34.87. **IDENTIFY:** The image formed by refraction at the surface of the eye is located by $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$.

SET UP: $n_a = 1.00$, $n_b = 1.35$. $R > 0$. For a distant object, $s \approx \infty$ and $\frac{1}{s} \approx 0$.

EXECUTE: **(a)** $s \approx \infty$ and $s' = 2.5\text{ cm}$: $\frac{1.35}{2.5\text{ cm}} = \frac{1.35 - 1.00}{R}$ and $R = 0.648\text{ cm} = 6.48\text{ mm}$.

(b) $R = 0.648\text{ cm}$ and $s = 25\text{ cm}$: $\frac{1.00}{25\text{ cm}} + \frac{1.35}{s'} = \frac{1.35 - 1.00}{0.648}$. $\frac{1.35}{s'} = 0.500$ and $s' = 2.70\text{ cm} = 27.0\text{ mm}$. The image is formed behind the retina.

(c) Calculate s' for $s \approx \infty$ and $R = 0.50\text{ cm}$: $\frac{1.35}{s'} = \frac{1.35 - 1.00}{0.50\text{ cm}}$. $s' = 1.93\text{ cm} = 19.3\text{ mm}$. The image is formed in front of the retina.

EVALUATE: The cornea alone cannot achieve focus of both close and distant objects.

34.88. **IDENTIFY:** Apply $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$ and $m = -\frac{n_a s'}{n_b s}$ to each surface. The overall magnification is $m = m_1 m_2$. The image formed by the first surface is the object for the second surface.

SET UP: For the first surface, $n_a = 1.00$, $n_b = 1.60$ and $R = +15.0\text{ cm}$. For the second surface, $n_a = 1.60$, $n_b = 1.00$ and $R \to \infty$.

EXECUTE: **(a)** $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \Rightarrow \frac{1}{12.0\text{ cm}} + \frac{1.60}{s'} = \frac{0.60}{15.0\text{ cm}} \Rightarrow s' = -36.9\text{ cm}$. The object distance for the far end of the rod is $50.0\text{ cm} - (-36.9\text{ cm}) = 86.9\text{ cm}$.

$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \Rightarrow \frac{1.60}{86.9\text{ cm}} + \frac{1}{s'} = 0 \Rightarrow s' = -54.3\text{ cm}$. The final image is 4.3 cm to the left of the vertex of the hemispherical surface.

(b) The magnification is the product of the two magnifications:

$$m_1 = -\frac{n_a s'}{n_b s} = -\frac{-36.9}{(1.60)(12.0)} = 1.92,\ m_2 = 1.00 \Rightarrow m = m_1 m_2 = 1.92.$$

EVALUATE: The final image is virtual, erect and larger than the object.

34.89. **IDENTIFY:** Apply $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$ to each surface. The image of the first surface is the object for the second surface. The relation between s_1' and s_2 involves the length *d* of the rod.

SET UP: For the first surface, $n_a=1.00$, $n_b=1.55$ and $R=+6.00\text{ cm}$. For the second surface, $n_a=1.55$, $n_b=1.00$ and $R=-6.00\text{ cm}$.

EXECUTE: We have images formed from both ends. From the first surface:

$$\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}\Rightarrow\frac{1}{25.0\text{ cm}}+\frac{1.55}{s'}=\frac{0.55}{6.00\text{ cm}}\Rightarrow s'=30.0\text{ cm}.$$

This image becomes the object for the second end:

$$\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}\Rightarrow\frac{1.55}{d-30.0\text{ cm}}+\frac{1}{65.0\text{ cm}}=\frac{-0.55}{-6.00\text{ cm}}.$$

$$d-30.0\text{ cm}=20.3\text{ cm}\Rightarrow d=50.3\text{ cm}.$$

EVALUATE: The final image is real. The first image is 20.3 cm to the left of the second surface and serves as a real object.

34.90. **IDENTIFY** and **SET UP:** Use $\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ to calculate the focal length of the lenses. The image formed by the first lens serves at the object for the second lens. $m_{\text{tot}}=m_1m_2$. $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ gives $s'=\frac{sf}{s-f}$.

EXECUTE: **(a)** $\frac{1}{f}=(0.60)\left(\frac{1}{12.0\text{ cm}}-\frac{1}{28.0\text{ cm}}\right)$ and $f=+35.0\text{ cm}$.

<u>Lens 1:</u> $f_1=+35.0\text{ cm}$. $s_1=+45.0\text{ cm}$. $s_1'=\frac{s_1f_1}{s_1-f_1}=\frac{(45.0\text{ cm})(35.0\text{ cm})}{45.0\text{ cm}-35.0\text{ cm}}=+158\text{ cm}$.

$m_1=-\frac{s_1'}{s_1}=-\frac{158\text{ cm}}{45.0\text{ cm}}=-3.51$. $|y_1'|=|m_1|y_1=(3.51)(5.00\text{ mm})=17.6\text{ mm}$. The image of the first lens is 158 cm to the right of lens 1 and is 17.6 mm tall.

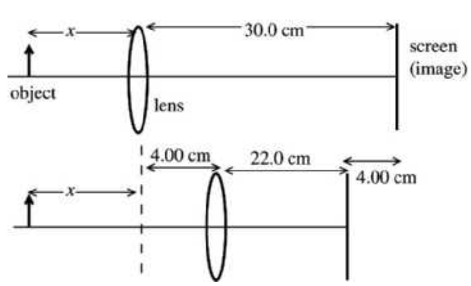
(b) The image of lens 1 is $315\text{ cm}-158\text{ cm}=157\text{ cm}$ to the left of lens 2. $f_2=+35.0\text{ cm}$. $s_2=+157\text{ cm}$.

$s_2'=\frac{s_2f_2}{s_2-f_2}=\frac{(157\text{ cm})(35.0\text{ cm})}{157\text{ cm}-35.0\text{ cm}}=+45.0\text{ cm}$. $m_2=-\frac{s_2'}{s_2}=-\frac{45.0\text{ cm}}{157\text{ cm}}=-0.287$.

$m_{\text{tot}}=m_1m_2=(-3.51)(-0.287)=+1.00$. The final image is 45.0 cm to the right of lens 2. The final image is 5.00 mm tall. $m_{\text{tot}}>0$ and the final image is erect.

EVALUATE: The final image is real. It is erect because each lens produces an inversion of the image, and two inversions return the image to the orientation of the object.

34.91. **IDENTIFY** and **SET UP:** Apply Eq. (34.16) for each lens position. The lens to screen distance in each case is the image distance. There are two unknowns, the original object distance *x* and the focal length *f* of the lens. But each lens position gives an equation, so there are two equations for these two unknowns. The object, lens and screen before and after the lens is moved are shown in Figure 34.91.



$s=x;\ s'=30.0\text{ cm}$

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

$$\frac{1}{x}+\frac{1}{30.0\text{ cm}}=\frac{1}{f}$$

Figure 34.91

$s = x + 4.00\text{ cm}; s' = 22.0\text{ cm}$

$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ gives $\frac{1}{x + 4.00\text{ cm}} + \frac{1}{22.0\text{ cm}} = \frac{1}{f}$

EXECUTE: Equate these two expressions for $1/f$:

$\frac{1}{x} + \frac{1}{30.0\text{ cm}} = \frac{1}{x + 4.00\text{ cm}} + \frac{1}{22.0\text{ cm}}$

$\frac{1}{x} - \frac{1}{x + 4.00\text{ cm}} = \frac{1}{22.0\text{ cm}} - \frac{1}{30.0\text{ cm}}$

$\frac{x + 4.00\text{ cm} - x}{x(x + 4.00\text{ cm})} = \frac{30.0 - 22.0}{660\text{ cm}}$ and $\frac{4.00\text{ cm}}{x(x + 4.00\text{ cm})} = \frac{8}{660\text{ cm}}$

$x^2 + (4.00\text{ cm})x - 330\text{ cm}^2 = 0$ and $x = \frac{1}{2}(-4.00 \pm \sqrt{16.0 + 4(330)})\text{ cm}$

x must be positive so $x = \frac{1}{2}(-4.00 + 36.55)\text{ cm} = 16.28\text{ cm}$

Then $\frac{1}{x} + \frac{1}{30.0\text{ cm}} = \frac{1}{f}$ and $\frac{1}{f} = \frac{1}{16.28\text{ cm}} + \frac{1}{30.0\text{ cm}}$

$f = +10.55\text{ cm}$, which rounds to 10.6 cm. $f > 0$; the lens is converging.

EVALUATE: We can check that $s = 16.28\text{ cm}$ and $f = 10.55\text{ cm}$ gives $s' = 30.0\text{ cm}$ and that $s = (16.28 + 4.0)\text{ cm} = 20.28\text{ cm}$ and $f = 10.55\text{ cm}$ gives $s' = 22.0\text{ cm}$.

34.92. **IDENTIFY:** Apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ and $m = -\frac{s'}{s}$.

SET UP: $s + s' = 18.0\text{ cm}$

EXECUTE: **(a)** $\frac{1}{18.0\text{ cm} - s'} + \frac{1}{s'} = \frac{1}{3.00\text{ cm}}$. $(s')^2 - (18.0\text{ cm})s' + 54.0\text{ cm}^2 = 0$ so $s' = 14.2\text{ cm}$ or 3.80 cm.

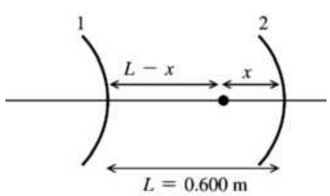
$s = 3.80\text{ cm}$ or 14.2 cm, so the lens must either be 3.80 cm or 14.2 cm from the object.

(b) $s = 3.80\text{ cm}: m = -\frac{s'}{s} = -\frac{14.2}{3.8} = -3.74$. $s = 14.2\text{ cm}: m = -\frac{s'}{s} = -\frac{3.8}{14.2} = -0.268$.

EVALUATE: Since the image is projected onto the screen, the image is real and s' is positive. We assumed this when we wrote the condition $s + s' = 18.0\text{ cm}$.

34.93. **(a) IDENTIFY:** Use Eq. (34.6) to locate the image formed by each mirror. The image formed by the first mirror serves as the object for the second mirror.

SET UP: The positions of the object and the two mirrors are shown in Figure 34.93a.



$|R| = 0.360\text{ m}$

$|f| = |R|/2 = 0.180\text{ m}$

Figure 34.93a

EXECUTE: Image formed by convex mirror (mirror #1):

convex means $f_1 = -0.180\text{ m}; s_1 = L - x$

$$s_1' = \frac{s_1 f_1}{s_1 - f_1} = \frac{(L - x)(-0.180\text{ m})}{L - x + 0.180\text{ m}} = -(0.180\text{ m})\left(\frac{0.600\text{ m} - x}{0.780\text{ m} - x}\right) < 0$$

The image is $(0.180\text{ m})\left(\dfrac{0.600\text{ m}-x}{0.780\text{ m}-x}\right)$ to the left of mirror #1 so is

$$0.600\text{ m}+(0.180\text{ m})\left(\frac{0.600\text{ m}-x}{0.780\text{ m}-x}\right)=\frac{0.576\text{ m}^2-(0.780\text{ m})x}{0.780\text{ m}-x}$$ to the left of mirror #2.

<u>Image formed by concave mirror (mirror #2);</u>

concave implies $f_2=+0.180\text{ m}$

$$s_2=\frac{0.576\text{ m}^2-(0.780\text{ m})x}{0.780\text{ m}-x}$$

Rays return to the source implies $s_2'=x$. Using these expressions in $s_2=\dfrac{s_2' f_2}{s_2'-f_2}$ gives

$$\frac{0.576\text{ m}^2-(0.780\text{ m})x}{0.780\text{ m}-x}=\frac{(0.180\text{ m})x}{x-0.180\text{ m}}$$

$$0.600x^2-(0.576\text{ m})x+0.10368\text{ m}^2=0$$

$$x=\frac{1}{1.20}\left(0.576\pm\sqrt{(0.576)^2-4(0.600)(0.10368)}\right)\text{ m}=\frac{1}{1.20}(0.576\pm0.288)\text{ m}$$

$x=0.72\text{ m}$ (impossible; can't have $x>L=0.600\text{ m}$) or $x=0.24\text{ m}.$

(b) SET UP: Which mirror is #1 and which is #2 is now reversed form part (a). This is shown in Figure 34.93b.

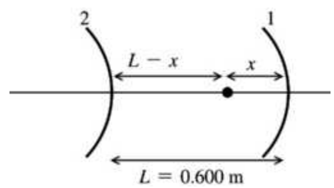


Figure 34.93b

EXECUTE: <u>Image formed by concave mirror (mirror #1):</u>

concave means $f_1=+0.180\text{ m}; s_1=x$

$$s_1'=\frac{s_1 f_1}{s_1-f_1}=\frac{(0.180\text{ m})x}{x-0.180\text{ m}}$$

The image is $\dfrac{(0.180\text{ m})x}{x-0.180\text{ m}}$ to the left of mirror #1, so

$$s_2=0.600\text{ m}-\frac{(0.180\text{ m})x}{x-0.180\text{ m}}=\frac{(0.420\text{ m})x-0.180\text{ m}^2}{x-0.180\text{ m}}$$

<u>Image formed by convex mirror (mirror #2):</u>

convex means $f_2=-0.180\text{ m}$

rays return to the source means $s_2'=L-x=0.600\text{ m}-x$

$\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}$ gives

$$\frac{x-0.180\text{ m}}{(0.420\text{ m})x-0.180\text{ m}^2}+\frac{1}{0.600\text{ m}-x}=-\frac{1}{0.180\text{ m}}$$

$$\frac{x-0.180\text{ m}}{(0.420\text{ m})x-0.180\text{ m}^2}=-\left(\frac{0.780\text{ m}-x}{0.180\text{ m}^2-(0.180\text{ m})x}\right)$$

$$0.600x^2-(0.576\text{ m})x+0.1036\text{ m}^2=0$$

This is the same quadratic equation as obtained in part (a), so again $x=0.24\text{ m}.$

EVALUATE: For $x = 0.24$ m the image is at the location of the source, both for rays that initially travel from the source toward the left and for rays that travel from the source toward the right.

34.94. IDENTIFY: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ gives $s'=\frac{sf}{s-f}$, for both the mirror and the lens.

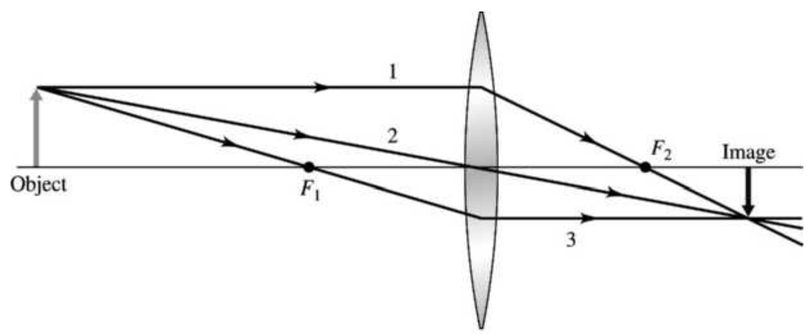
SET UP: For the second image, the image formed by the mirror serves as the object for the lens. For the mirror, $f_m = +10.0$ cm. For the lens, $f = 32.0$ cm. The center of curvature of the mirror is $R = 2f_m = 20.0$ cm to the right of the mirror vertex.

EXECUTE: **(a)** The principal-ray diagrams from the two images are sketched in Figures 34.94a–b. In Figure 34.94b, only the image formed by the mirror is shown. This image is at the location of the candle so the principal-ray diagram that shows the image formation when the image of the mirror serves as the object for the lens is analogous to that in Figure 34.94a and is not drawn.

(b) Image formed by the light that passes directly through the lens: The candle is 85.0 cm to the left of the lens. $s'=\frac{sf}{s-f}=\frac{(85.0\text{ cm})(32.0\text{ cm})}{85.0\text{ cm}-32.0\text{ cm}}=+51.3\text{ cm}$. $m=-\frac{s'}{s}=-\frac{51.3\text{ cm}}{85.0\text{ cm}}=-0.604$. This image is 51.3 cm to the right of the lens. $s'>0$ so the image is real. $m<0$ so the image is inverted. Image formed by the light that first reflects off the mirror: First consider the image formed by the mirror. The candle is 20.0 cm to the right of the mirror, so $s = +20.0$ cm. $s'=\frac{sf}{s-f}=\frac{(20.0\text{ cm})(10.0\text{ cm})}{20.0\text{ cm}-10.0\text{ cm}}=20.0\text{ cm}$.

$m_1=-\frac{s_1'}{s_1}=-\frac{20.0\text{ cm}}{20.0\text{ cm}}=-1.00$. The image formed by the mirror is at the location of the candle, so $s_2 = +85.0$ cm and $s_2' = 51.3$ cm. $m_2 = -0.604$. $m_{tot} = m_1 m_2 = (-1.00)(-0.604) = 0.604$. The second image is 51.3 cm to the right of the lens. $s_2' > 0$, so the final image is real. $m_{tot} > 0$, so the final image is erect.

EVALUATE: The two images are at the same place. They are the same size. One is erect and one is inverted.



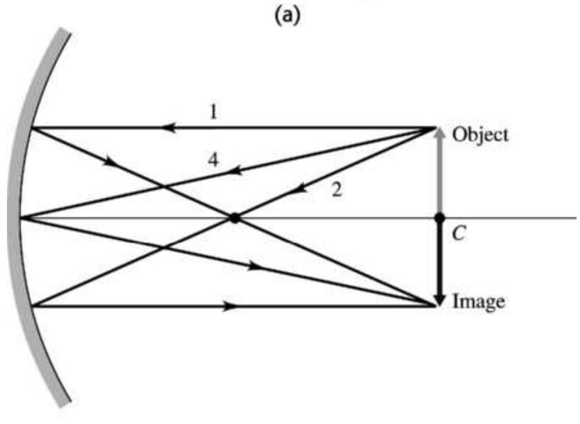


Figure 34.94

34.95. **IDENTIFY:** Apply $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$ to each case.

SET UP: $s=20.0$ cm. $R>0$. Use $s'=+9.12$ cm to find *R*. For this calculation, $n_a=1.00$ and $n_b=1.55$. Then repeat the calculation with $n_a=1.33$.

EXECUTE: $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$ gives $\frac{1.00}{20.0\text{ cm}}+\frac{1.55}{9.12\text{ cm}}=\frac{1.55-1.00}{R}$. $R=2.50$ cm.

Then $\frac{1.33}{20.0\text{ cm}}+\frac{1.55}{s'}=\frac{1.55-1.33}{2.50\text{ cm}}$ gives $s'=72.1$ cm. The image is 72.1 cm to the right of the surface vertex.

EVALUATE: With the rod in air the image is real and with the rod in water the image is also real.

34.96. **IDENTIFY:** Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ to each lens. The image formed by the first lens serves as the object for the second lens. The focal length of the lens combination is defined by $\frac{1}{s_1}+\frac{1}{s_2'}=\frac{1}{f}$. In part (b) use $\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ to calculate *f* for the meniscus lens and for the CCl_4, treated as a thin lens.

SET UP: With two lenses of different focal length in contact, the image distance from the first lens becomes exactly minus the object distance for the second lens.

EXECUTE: **(a)** $\frac{1}{s_1}+\frac{1}{s_1'}=\frac{1}{f_1}\Rightarrow\frac{1}{s_1'}=\frac{1}{f_1}-\frac{1}{s_1}$ and $\frac{1}{s_2}+\frac{1}{s_2'}=\frac{1}{-s_1'}+\frac{1}{s_2'}=\left(\frac{1}{s_1}-\frac{1}{f_1}\right)+\frac{1}{s_2'}=\frac{1}{f_2}$. But overall for the lens system, $\frac{1}{s_1}+\frac{1}{s_2'}=\frac{1}{f}\Rightarrow\frac{1}{f}=\frac{1}{f_2}+\frac{1}{f_1}$.

(b) With carbon tetrachloride sitting in a meniscus lens, we have two lenses in contact. All we need in order to calculate the system's focal length is calculate the individual focal lengths, and then use the formula from part (a). For the meniscus lens

$$\frac{1}{f_{\text{m}}}=(n_b-n_a)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=(0.55)\left(\frac{1}{4.50\text{ cm}}-\frac{1}{9.00\text{ cm}}\right)=0.061\text{ cm}^{-1}\text{ and }f_{\text{m}}=16.4\text{ cm.}$$

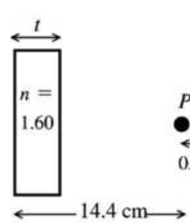
For the CCl_4: $\frac{1}{f_{\text{w}}}=(n_b-n_a)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=(0.46)\left(\frac{1}{9.00\text{ cm}}=\frac{1}{\infty}\right)=0.051\text{ cm}^{-1}$ and $f_{\text{w}}=19.6$ cm.

$$\frac{1}{f}=\frac{1}{f_{\text{w}}}+\frac{1}{f_{\text{m}}}=0.112\text{ cm}^{-1}\text{ and }f=8.93\text{ cm.}$$

EVALUATE: $f=\frac{f_1f_2}{f_1+f_2}$, so *f* for the combination is less than either f_1 or f_2.

34.97. **IDENTIFY:** Apply Eq. (34.11) with $R\to\infty$ to the refraction at each surface. For refraction at the first surface the point *P* serves as a virtual object. The image formed by the first refraction serves as the object for the second refraction.

SET UP: The glass plate and the two points are shown in Figure 34.97.



plane faces means $R\to\infty$ and

$$\frac{n_a}{s}+\frac{n_b}{s'}=0$$

$$s'=-\frac{n_b}{n_a}s$$

Figure 34.97

EXECUTE: refraction at the first (left-hand) surface of the piece of glass:
The rays converging toward point P constitute a virtual object for this surface, so $s = -14.4$ cm.
$n_a = 1.00$, $n_b = 1.60$.

$$s' = -\frac{1.60}{1.00}(-14.4 \text{ cm}) = +23.0 \text{ cm}$$

This image is 23.0 cm to the right of the first surface so is a distance $23.0 \text{ cm} - t$ to the right of the second surface. This image serves as a virtual object for the second surface.
refraction at the second (right-hand) surface of the piece of glass:
The image is at P' so $s' = 14.4 \text{ cm} + 0.30 \text{ cm} - t = 14.7 \text{ cm} - t$. $s = -(23.0 \text{ cm} - t)$; $n_a = 1.60$; $n_b = 1.00$

$$s' = -\frac{n_b}{n_a}s \text{ gives } 14.7 \text{ cm} - t = -\left(\frac{1.00}{1.60}\right)(-[23.0 \text{ cm} - t]).\ \ 14.7 \text{ cm} - t = +14.4 \text{ cm} - 0.625t.$$

$0.375t = 0.30$ cm and $t = 0.80$ cm

EVALUATE: The overall effect of the piece of glass is to diverge the rays and move their convergence point to the right. For a real object, refraction at a plane surface always produces a virtual image, but with a virtual object the image can be real.

34.98. **IDENTIFY:** Apply the two equations $\frac{n_a}{s_1} + \frac{n_b}{s_1'} = \frac{n_b - n_a}{R_1}$ and $\frac{n_b}{s_2} + \frac{n_c}{s_2'} = \frac{n_c - n_b}{R_2}$.

SET UP: $n_a = n_{\text{liq}} = n_c$, $n_b = n$, and $s_1' = -s_2$.

EXECUTE: **(a)** $\frac{n_{\text{liq}}}{s_1} + \frac{n}{s_1'} = \frac{n - n_{\text{liq}}}{R_1}$ and $\frac{n}{-s_1'} + \frac{n_{\text{liq}}}{s_2'} = \frac{n_{\text{liq}} - n}{R_2}$. $\frac{1}{s_1} + \frac{1}{s_2'} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{f'} = (n/n_{\text{liq}} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.

(b) Comparing the equations for focal length in and out of air we have:

$$f(n-1) = f'(n/n_{\text{liq}} - 1) = f'\left(\frac{n - n_{\text{liq}}}{n_{\text{liq}}}\right) \Rightarrow f' = \left[\frac{n_{\text{liq}}(n-1)}{n - n_{\text{liq}}}\right]f.$$

EVALUATE: When $n_{\text{liq}} = 1$, $f' = f$, as it should.

34.99. **IDENTIFY:** Apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$.

SET UP: The image formed by the converging lens is 30.0 cm from the converging lens, and becomes a virtual object for the diverging lens at a position 15.0 cm to the right of the diverging lens. The final image is projected $15 \text{ cm} + 19.2 \text{ cm} = 34.2 \text{ cm}$ from the diverging lens.

EXECUTE: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{-15.0 \text{ cm}} + \frac{1}{34.2 \text{ cm}} = \frac{1}{f} \Rightarrow f = -26.7$ cm.

EVALUATE: Our calculation yields a negative value of f, which should be the case for a diverging lens.

34.100. **IDENTIFY:** The spherical mirror forms an image of the object. It forms another image when the image of the plane mirror serves as an object.

SET UP: For the convex mirror $f = -24.0$ cm. The image formed by the plane mirror is 10.0 cm to the right of the plane mirror, so is $20.0 \text{ cm} + 10.0 \text{ cm} = 30.0 \text{ cm}$ from the vertex of the spherical mirror.

EXECUTE: The first image formed by the spherical mirror is the one where the light immediately strikes its surface, without bouncing from the plane mirror.

$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{10.0 \text{ cm}} + \frac{1}{s'} = \frac{1}{-24.0 \text{ cm}} \Rightarrow s' = -7.06$ cm, and the image height is

$$y' = -\frac{s'}{s}y = -\frac{-7.06}{10.0}(0.250 \text{ cm}) = 0.177 \text{ cm}.$$

The second image of the plane mirror image is located 30.0 cm from the vertex of the spherical mirror.

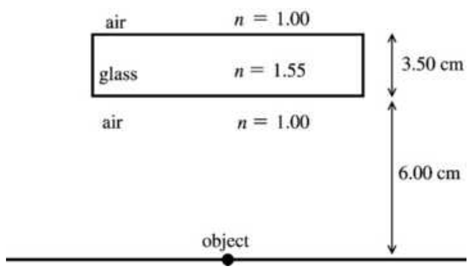
$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{30.0 \text{ cm}} + \frac{1}{s'} = \frac{1}{-24.0 \text{ cm}} \Rightarrow s' = -13.3$ cm and the image height is

$$y' = -\frac{s'}{s}y = -\frac{-13.3}{30.0}(0.250 \text{ cm}) = 0.111 \text{ cm}.$$

Evaluate: Other images are formed by additional reflections from the two mirrors.

34.101. Identify: In the sketch in Figure 34.101 the light travels upward from the object. Apply Eq. (34.11) with $R \to \infty$ to the refraction at each surface. The image formed by the first surface serves as the object for the second surface.

Set Up: The locations of the object and the glass plate are shown in Figure 34.101.



For a plane (flat) surface

$R \to \infty$ so $\dfrac{n_a}{s} + \dfrac{n_b}{s'} = 0$

$s' = -\dfrac{n_b}{n_a} s$

Figure 34.101

Execute: <u>First refraction (air → glass):</u>

$n_a = 1.00$; $n_b = 1.55$; $s = 6.00$ cm

$$s' = -\frac{n_b}{n_a} s = -\frac{1.55}{1.00}(6.00\text{ cm}) = -9.30\text{ cm}.$$

The image is 9.30 cm below the lower surface of the glass, so is $9.30\text{ cm} + 3.50\text{ cm} = 12.8\text{ cm}$ below the upper surface.

<u>Second refraction (glass → air):</u>

$n_a = 1.55$; $n_b = 1.00$; $s = +12.8$ cm

$$s' = -\frac{n_b}{n_a} s = -\frac{1.00}{1.55}(12.8\text{ cm}) = -8.26\text{ cm}$$

The image of the page is 8.26 cm below the top surface of the glass plate and therefore $9.50\text{ cm} - 8.26\text{ cm} = 1.24\text{ cm}$ above the page.

Evaluate: The image is virtual. If you view the object by looking down from above the plate, the image of the page that you see is closer to your eye than the page is.

34.102. Identify: Light refracts at the front surface of the lens, refracts at the glass-water interface, reflects from the plane mirror and passes through the two interfaces again, now traveling in the opposite direction.

Set Up: Use the focal length in air to find the radius of curvature R of the lens surfaces.

Execute: **(a)** $\dfrac{1}{f} = (n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right) \Rightarrow \dfrac{1}{40\text{ cm}} = 0.52\left(\dfrac{2}{R}\right) \Rightarrow R = 41.6\text{ cm}.$

At the air–lens interface: $\dfrac{n_a}{s} + \dfrac{n_b}{s'} = \dfrac{n_b - n_a}{R} \Rightarrow \dfrac{1}{70.0\text{ cm}} + \dfrac{1.52}{s_1'} = \dfrac{0.52}{41.6\text{ cm}}$ and

$s_1' = -851$ cm and $s_2 = 851$ cm.

At the lens–water interface: $\Rightarrow \dfrac{1.52}{851\text{ cm}} + \dfrac{1.33}{s_2'} = \dfrac{-0.187}{-41.6\text{ cm}}$ and $s_2' = 491\text{ cm}.$

The mirror reflects the image back (since there is just 90 cm between the lens and mirror.) So, the position of the image is 401 cm to the left of the mirror, or 311 cm to the left of the lens.

At the water–lens interface: $\Rightarrow \dfrac{1.33}{-311\text{ cm}} + \dfrac{1.52}{s_3'} = \dfrac{0.187}{41.6\text{ cm}}$ and $s_3' = +173\text{ cm}.$

At the lens–air interface: $\Rightarrow \dfrac{1.52}{-173\text{ cm}} + \dfrac{1}{s_4'} = \dfrac{-0.52}{-41.6\text{ cm}}$ and $s_4' = +47.0\text{ cm},$ to the left of the lens.

$$m = m_1 m_2 m_3 m_4 = \left(\frac{n_{a1}s_1'}{n_{b1}s_1}\right)\left(\frac{n_{a2}s_2'}{n_{b2}s_2}\right)\left(\frac{n_{a3}s_3'}{n_{b3}s_3}\right)\left(\frac{n_{a4}s_4'}{n_{b4}s_4}\right) = \left(\frac{-851}{70}\right)\left(\frac{491}{851}\right)\left(\frac{+173}{-311}\right)\left(\frac{+47.0}{-173}\right) = -1.06.$$

(Note all the indices of refraction cancel out.)
(b) The image is real.
(c) The image is inverted.
(d) The final height is $y' = my = (1.06)(4.00 \text{ mm}) = 4.24 \text{ mm}$.

EVALUATE: The final image is real even though it is on the same side of the lens as the object!

34.103. IDENTIFY: The camera lens can be modeled as a thin lens that forms an image on the film.

SET UP: The thin-lens equation is $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$, and the magnification of the lens is $m=-\frac{s'}{s}$.

EXECUTE: **(a)** $m=-\frac{s'}{s}=\frac{y'}{y}=\frac{1}{4}\frac{(0.0360 \text{ m})}{(12.0 \text{ m})} \Rightarrow s'=(7.50\times10^{-4})\,s$,

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{s}+\frac{1}{(7.50\times10^{-4})s}=\frac{1}{s}\left(1+\frac{1}{7.50\times10^{-4}}\right)=\frac{1}{f}=\frac{1}{0.0350 \text{ m}} \Rightarrow s=46.7 \text{ m}.$$

(b) To just fill the frame, the magnification must be 3.00×10^{-3} so:

$$\frac{1}{s}\left(1+\frac{1}{3.00\times10^{-3}}\right)=\frac{1}{f}=\frac{1}{0.0350 \text{ m}} \Rightarrow s=11.7 \text{ m}.$$

Since the boat is originally 46.7 m away, the distance you must move closer to the boat is $46.7 \text{ m} - 11.7 \text{ m} = 35.0 \text{ m}$.

EVALUATE: This result seems to imply that if you are 4 times as far, the image is ¼ as large on the film. However, this result is only an approximation, and would not be true for very close distances. It is a better approximation for large distances.

34.104. IDENTIFY: The smallest image we can resolve occurs when the image is the size of a retinal cell.

SET UP: $m=-\frac{s'}{s}=\frac{y'}{y}$. $s'=2.50$ cm.

$|y'|=5.0\ \mu\text{m}$. The angle subtended (in radians) is height divided by distance from the eye.

EXECUTE: **(a)** $m=-\frac{s'}{s}=-\frac{2.50 \text{ cm}}{25 \text{ cm}}=-0.10$. $y=\left|\frac{y'}{m}\right|=\frac{5.0\ \mu\text{m}}{0.10}=50\ \mu\text{m}$.

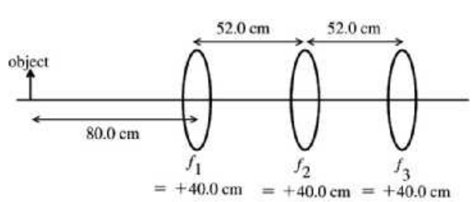
(b) $\theta=\frac{y}{s}=\frac{50\ \mu\text{m}}{25 \text{ cm}}=\frac{50\times10^{-6} \text{ m}}{25\times10^{-2} \text{ m}}=2.0\times10^{-4} \text{ rad}=0.0115°=0.69 \text{ min}$. This is only a bit smaller than the typical experimental value of 1.0 min.

EVALUATE: The angle subtended by the object equals the angular size of the image,

$$\frac{|y'|}{s'}=\frac{5.0\times10^{-6} \text{ m}}{2.50\times10^{-2} \text{ m}}=2.0\times10^{-4} \text{ rad}.$$

34.105. IDENTIFY: Apply Eq. (34.16) to calculate the image distance for each lens. The image formed by the first lens serves as the object for the second lens, and the image formed by the second lens serves as the object for the third lens.

SET UP: The positions of the object and lenses are shown in Figure 34.105.



$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

$$\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}=\frac{s-f}{sf}$$

$$s'=\frac{sf}{s-f}$$

Figure 34.105

EXECUTE: <u>lens #1</u>

$s = +80.0\text{ cm};\ f = +40.0\text{ cm}$

$$s' = \frac{sf}{s-f} = \frac{(+80.0\text{ cm})(+40.0\text{ cm})}{+80.0\text{ cm} - 40.0\text{ cm}} = +80.0\text{ cm}$$

The image formed by the first lens is 80.0 cm to the right of the first lens, so it is $80.0\text{ cm} - 52.0\text{ cm} = 28.0\text{ cm}$ to the right of the second lens.

<u>lens #2</u>

$s = -28.0\text{ cm};\ f = +40.0\text{ cm}$

$$s' = \frac{sf}{s-f} = \frac{(-28.0\text{ cm})(+40.0\text{ cm})}{-28.0\text{ cm} - 40.0\text{ cm}} = +16.47\text{ cm}$$

The image formed by the second lens is 16.47 cm to the right of the second lens, so it is $52.0\text{ cm} - 16.47\text{ cm} = 35.53\text{ cm}$ to the left of the third lens.

<u>lens #3</u>

$s = +35.53\text{ cm};\ f = +40.0\text{ cm}$

$$s' = \frac{sf}{s-f} = \frac{(+35.53\text{ cm})(+40.0\text{ cm})}{+35.53\text{ cm} - 40.0\text{ cm}} = -318\text{ cm}$$

The final image is 318 cm to the left of the third lens, so it is $318\text{ cm} - 52\text{ cm} - 52\text{ cm} - 80\text{ cm} = 134\text{ cm}$ to the left of the object.

EVALUATE: We used the separation between the lenses and the sign conventions for s and s' to determine the object distances for the second and third lenses. The final image is virtual since the final s' is negative.

34.106. **IDENTIFY:** Apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ and calculate s' for each s.

SET UP: $f = 90\text{ mm}$

EXECUTE: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{1300\text{ mm}} + \frac{1}{s'} = \frac{1}{90\text{ mm}} \Rightarrow s' = 96.7\text{ mm}.$

$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{6500\text{ mm}} + \frac{1}{s'} = \frac{1}{90\text{ mm}} \Rightarrow s' = 91.3\text{ mm}.$

$\Rightarrow \Delta s' = 96.7\text{ mm} - 91.3\text{ mm} = 5.4\text{ mm}$ toward the film

EVALUATE: $s' = \frac{sf}{s-f}$. For $f > 0$ and $s > f$, s' decreases as s increases.

34.107. **IDENTIFY** and **SET UP:** The generalization of Eq. (34.22) is $M = \frac{\text{near point}}{f}$, so $f = \frac{\text{near point}}{M}$.

EXECUTE: **(a)** age 10, near point = 7 cm

$f = \frac{7\text{ cm}}{2.0} = 3.5\text{ cm}$

(b) age 30, near point = 14 cm

$f = \frac{14\text{ cm}}{2.0} = 7.0\text{ cm}$

(c) age 60, near point = 200 cm

$f = \frac{200\text{ cm}}{2.0} = 100\text{ cm}$

(d) $f = 3.5\text{ cm}$ (from part (a)) and near point = 200 cm (for 60-year-old)

$M = \frac{200\text{ cm}}{3.5\text{ cm}} = 57$

(e) EVALUATE: No. The reason $f = 3.5\text{ cm}$ gives a larger M for a 60-year-old than for a 10-year-old is that the eye of the older person can't focus on as close an object as the younger person can. The unaided eye of the 60-year-old must view a much smaller angular size, and that is why the same f gives a much larger M. The angular size of the image depends only on f and is the same for the two ages.

34.108. **IDENTIFY:** Use $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ to calculate *s* that gives $s'=-25$ cm. $M=\frac{\theta'}{\theta}$.

SET UP: Let the height of the object be *y*, so $\theta'=\frac{y}{s}$ and $\theta=\frac{y}{25\text{ cm}}$.

EXECUTE: **(a)** $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{s}+\frac{1}{-25\text{ cm}}=\frac{1}{f}\Rightarrow s=\frac{f(25\text{ cm})}{f+25\text{ cm}}$.

(b) $\theta'=\arctan\left(\frac{y}{s}\right)=\arctan\left(\frac{y(f+25\text{ cm})}{f(25\text{ cm})}\right)\approx\frac{y(f+25\text{ cm})}{f(25\text{ cm})}$.

(c) $M=\frac{\theta'}{\theta}=\frac{y(f+25\text{ cm})}{f(25\text{ cm})}\frac{1}{y/25\text{ cm}}=\frac{f+25\text{ cm}}{f}$.

(d) If $f=10\text{ cm}\Rightarrow M=\frac{10\text{ cm}+25\text{ cm}}{10\text{ cm}}=3.5$. This is 1.4 times greater than the magnification obtained if the image if formed at infinity $\left(M_\infty=\frac{25\text{ cm}}{f}=2.5\right)$.

EVALUATE: **(e)** Having the first image form just within the focal length puts one in the situation described above, where it acts as a source that yields an enlarged virtual image. If the first image fell just outside the second focal point, then the image would be real and diminished.

34.109. **IDENTIFY:** Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$. The near point is at infinity, so that is where the image must be formed for any objects that are close.

SET UP: The power in diopters equals $\frac{1}{f}$, with *f* in meters.

EXECUTE: $\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}=\frac{1}{24\text{ cm}}+\frac{1}{-\infty}=\frac{1}{0.24\text{ m}}=4.17$ diopters.

EVALUATE: To focus on closer objects, the power must be increased.

34.110. **IDENTIFY:** Apply $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$.

SET UP: $n_a=1.00$, $n_b=1.40$.

EXECUTE: $\frac{1}{36.0\text{ cm}}+\frac{1.40}{s'}=\frac{0.40}{0.75\text{ cm}}\Rightarrow s'=2.77$ cm.

EVALUATE: This distance is greater than for the normal eye, which has a cornea vertex to retina distance of about 2.6 cm.

34.111. **IDENTIFY** and **SET UP:** The person's eye cannot focus on anything closer than 85.0 cm. The problem asks us to find the location of an object such that his old lenses produce a virtual image 85.0 cm from his eye. $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$. $P(\text{in diopters})=1/f\,(\text{in m})$.

EXECUTE: **(a)** $\frac{1}{f}=2.25$ diopters so $f=44.4$ cm. The image is 85.0 cm from his eye so is 83.0 cm from the eyeglass lens. Solving $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ for *s* gives $s=\frac{s'f}{s'-f}=\frac{(-83.0\text{ cm})(44.4\text{ cm})}{-83.0\text{ cm}-44.4\text{ cm}}=+28.9$ cm. The object is 28.9 cm from the eyeglasses so is 30.9 cm from his eyes.

(b) Now $s'=-85.0$ cm. $s=\frac{s'f}{s'-f}=\frac{(-85.0\text{ cm})(44.4\text{ cm})}{-85.0\text{ cm}-44.4\text{ cm}}=+29.2$ cm.

EVALUATE: The old glasses allow him to focus on objects as close as about 30 cm from his eyes. This is much better than a closest distance of 85 cm with no glasses, but his current glasses probably allow him to focus as close as 25 cm.

34.112. **IDENTIFY:** For u and u' as defined in Figure P34.112 in the textbook, $M = \frac{u'}{u}$.

SET UP: f_2 is negative. From Figure P34.112 in the textbook, the length of the telescope is $f_1 + f_2$, since f_2 is negative.

EXECUTE: **(a)** From the figure, $u = \frac{y}{f_1}$ and $u' = \frac{y}{|f_2|} = -\frac{y}{f_2}$. The angular magnification is $M = \frac{u'}{u} = -\frac{f_1}{f_2}$.

(b) $M = -\frac{f_1}{f_2} \Rightarrow f_2 = -\frac{f_1}{M} = -\frac{95.0\text{ cm}}{6.33} = -15.0\text{ cm}$.

(c) The length of the telescope is $95.0\text{ cm} - 15.0\text{ cm} = 80.0\text{ cm}$, compared to the length of 110 cm for the telescope in Exercise 34.65.

EVALUATE: An advantage of this construction is that the telescope is somewhat shorter.

34.113. **IDENTIFY:** Use similar triangles in Figure P34.113 in the textbook and Eq. (34.16) to derive the expressions called for in the problem.

(a) SET UP: The effect of the converging lens on the ray bundle is sketched in Figure 34.113a.

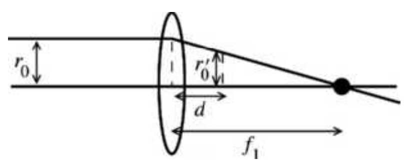


Figure 34.113a

EXECUTE: From similar triangles in Figure 34.113a, $\frac{r_0}{f_1} = \frac{r_0'}{f_1 - d}$.

Thus $r_0' = \left(\frac{f_1 - d}{f_1}\right) r_0$, as was to be shown.

(b) SET UP: The image at the focal point of the first lens, a distance f_1 to the right of the first lens, serves as the object for the second lens. The image is a distance $f_1 - d$ to the right of the second lens, so $s_2 = -(f_1 - d) = d - f_1$.

EXECUTE: $s_2' = \frac{s_2 f_2}{s_2 - f_2} = \frac{(d - f_1) f_2}{d - f_1 - f_2}$

$f_2 < 0$ so $|f_2| = -f_2$ and $s_2' = \frac{(f_1 - d)|f_2|}{|f_2| - f_1 + d}$, as was to be shown.

(c) SET UP: The effect of the diverging lens on the ray bundle is sketched in Figure 34.113b.

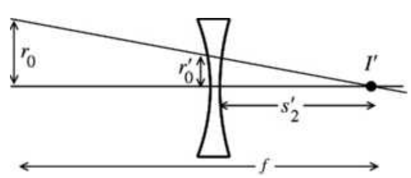


Figure 34.113b

EXECUTE: From similar triangles in the sketch, $\frac{r_0}{f} = \frac{r_0'}{s_2'}$.

Thus $\frac{r_0}{r_0'} = \frac{f}{s_2'}$.

From the results of part (a), $\frac{r_0}{r_0'} = \frac{f_1}{f_1 - d}$. Combining the two results gives $\frac{f_1}{f_1 - d} = \frac{f}{s_2'}$.

$$f = s_2'\left(\frac{f_1}{f_1 - d}\right) = \frac{(f_1 - d)|f_2| f_1}{(|f_2| - f_1 + d)(f_1 - d)} = \frac{f_1 |f_2|}{|f_2| - f_1 + d},$$ as was to be shown.

(d) SET UP: Put the numerical values into the expression derived in part (c).

EXECUTE: $f = \dfrac{f_1|f_2|}{|f_2| - f_1 + d}$

$f_1 = 12.0 \text{ cm}, \ |f_2| = 18.0 \text{ cm, so } f = \dfrac{216 \text{ cm}^2}{6.0 \text{ cm} + d}$

$d = 0$ gives $f = 36.0$ cm; maximum f

$d = 4.0$ cm gives $f = 21.6$ cm; minimum f

$f = 30.0$ cm says $30.0 \text{ cm} = \dfrac{216 \text{ cm}^2}{6.0 \text{ cm} + d}$

$6.0 \text{ cm} + d = 7.2$ cm and $d = 1.2$ cm

EVALUATE: Changing d produces a range of effective focal lengths. The effective focal length can be both smaller and larger than $f_1 + |f_2|$.

34.114. IDENTIFY: $|M| = \dfrac{\theta'}{\theta}$. $\theta = \left|\dfrac{y_1'}{f_1}\right|$, and $\theta' = \left|\dfrac{y_2'}{s_2'}\right|$. This gives $|M| = \left|\dfrac{y_2'}{s_2'} \cdot \dfrac{f_1}{y'_1}\right|$.

SET UP: Since the image formed by the objective is used as the object for the eyepiece, $y_1' = y_2$.

EXECUTE: $|M| = \left|\dfrac{y_2'}{s_2'} \cdot \dfrac{f_1}{y_2}\right| = \left|\dfrac{y_2'}{y_2} \cdot \dfrac{f_1}{s_2'}\right| = \left|\dfrac{s_2'}{s_2} \cdot \dfrac{f_1}{s_2'}\right| = \left|\dfrac{f_1}{s_2}\right|$. Therefore, $s_2 = \dfrac{f_1}{|M|} = \dfrac{48.0 \text{ cm}}{36} = 1.33 \text{ cm}$, and this is just outside the eyepiece focal point.

Now the distance from the mirror vertex to the lens is $f_1 + s_2 = 49.3$ cm, and so $\dfrac{1}{s_2} + \dfrac{1}{s_2'} = \dfrac{1}{f_2} \Rightarrow$

$s_2' = \left(\dfrac{1}{1.20 \text{ cm}} - \dfrac{1}{1.33 \text{ cm}}\right)^{-1} = 12.3$ cm. Thus we have a final image which is real and 12.3 cm from the eyepiece. (Take care to carry plenty of figures in the calculation because two close numbers are subtracted.)

EVALUATE: Eq. (34.25) gives $|M| = 40$, somewhat larger than $|M|$ for this telescope.

34.115. IDENTIFY and SET UP: The image formed by the objective is the object for the eyepiece. The total lateral magnification is $m_{\text{tot}} = m_1 m_2$. $f_1 = 8.00$ mm (objective); $f_2 = 7.50$ cm (eyepiece)

(a) The locations of the object, lenses and screen are shown in Figure 34.115.

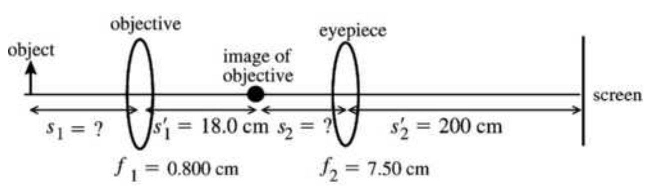


Figure 34.115

EXECUTE: Find the object distance s_1 for the objective:

$s_1' = +18.0 \text{ cm}, \ f_1 = 0.800 \text{ cm}, \ s_1 = ?$

$\dfrac{1}{s_1} + \dfrac{1}{s_1'} = \dfrac{1}{f_1}$, so $\dfrac{1}{s_1} = \dfrac{1}{f_1} - \dfrac{1}{s_1'} = \dfrac{s_1' - f_1}{s_1' f_1}$

$s_1 = \dfrac{s_1' f_1}{s_1' - f_1} = \dfrac{(18.0 \text{ cm})(0.800 \text{ cm})}{18.0 \text{ cm} - 0.800 \text{ cm}} = 0.8372 \text{ cm}$

Find the object distance s_2 for the eyepiece:

$s_2' = +200\text{ cm},\ f_2 = 7.50\text{ cm},\ s_2 = ?$

$$\frac{1}{s_2} + \frac{1}{s_2'} = \frac{1}{f_2}$$

$$s_2 = \frac{s_2' f_2}{s_2' - f_2} = \frac{(200\text{ cm})(7.50\text{ cm})}{200\text{ cm} - 7.50\text{ cm}} = 7.792\text{ cm}$$

Now we calculate the magnification for each lens:

$$m_1 = -\frac{s_1'}{s_1} = -\frac{18.0\text{ cm}}{0.8372\text{ cm}} = -21.50$$

$$m_2 = -\frac{s_2'}{s_2} = -\frac{200\text{ cm}}{7.792\text{ cm}} = -25.67$$

$m_{\text{tot}} = m_1 m_2 = (-21.50)(-25.67) = 552.$

(b) From the sketch we can see that the distance between the two lenses is $s_1' + s_2 = 18.0\text{ cm} + 7.792\text{ cm} = 25.8\text{ cm}.$

EVALUATE: The microscope is not being used in the conventional way; it merely serves as a two-lens system. In particular, the final image formed by the eyepiece in the problem is real, not virtual as is the case normally for a microscope. Eq. (34.24) does not apply here, and in any event gives the angular not the lateral magnification.

34.116. **IDENTIFY** and **SET UP:** Consider the ray diagram drawn in Figure 34.116.

EXECUTE: **(a)** Using the diagram and law of sines, $\dfrac{\sin\theta}{(R-f)} = \dfrac{\sin\alpha}{g}$ but $\sin\theta = \dfrac{h}{R} = \sin\alpha$ (law of reflection), and $g = (R - f)$. Bisecting the triangle: $\cos\theta = \dfrac{R/2}{(R-f)} \Rightarrow R\cos\theta - f\cos\theta = \dfrac{R}{2}.$

$f = \dfrac{R}{2}\left[2 - \dfrac{1}{\cos\theta}\right] = f_0\left[2 - \dfrac{1}{\cos\theta}\right].$ $f_0 = \dfrac{R}{2}$ is the value of f for θ near zero (incident ray near the axis). When θ increases, $(2 - 1/\cos\theta)$ decreases and f decreases.

(b) $\dfrac{f - f_0}{f_0} = -0.02 \Rightarrow \dfrac{f}{f_0} = 0.98$ so $2 - \dfrac{1}{\cos\theta} = 0.98.$ $\cos\theta = \dfrac{1}{2 - 0.98} = 0.98$ and $\theta = 11.4°.$

EVALUATE: For $\theta = 45°$, $f = 0.586 f_0$, and f approaches zero as θ approaches $60°$.

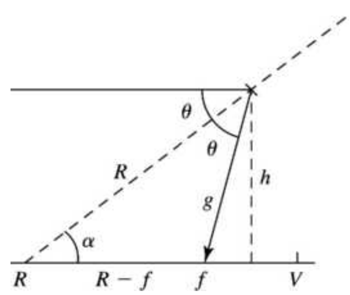


Figure 34.116

34.117. **IDENTIFY:** The distance between image and object can be calculated by taking the derivative of the separation distance and minimizing it.

SET UP: For a real image $s' > 0$ and the distance between the object and the image is $D = s + s'$. For a real image must have $s > f$.

EXECUTE: **(a)** $D = s + s'$ but $s' = \dfrac{sf}{s-f} \Rightarrow D = s + \dfrac{sf}{s-f} = \dfrac{s^2}{s-f}.$

$\frac{dD}{ds}=\frac{d}{ds}\left(\frac{s^2}{s-f}\right)=\frac{2s}{s-f}-\frac{s^2}{(s-f)^2}=\frac{s^2-2sf}{(s-f)^2}=0.$ $s^2-2sf=0.$ $s(s-2f)=0.$ $s=2f$ is the solution for which $s>f$. For $s=2f$, $s'=2f$. Therefore, the minimum separation is $2f+2f=4f$.

(b) A graph of D/f versus s/f is sketched in Figure 34.117. Note that the minimum does occur for $D=4f$.

EVALUATE: If, for example, $s=3f/2$, then $s'=3f$ and $D=s+s'=4.5f$, greater than the minimum value.

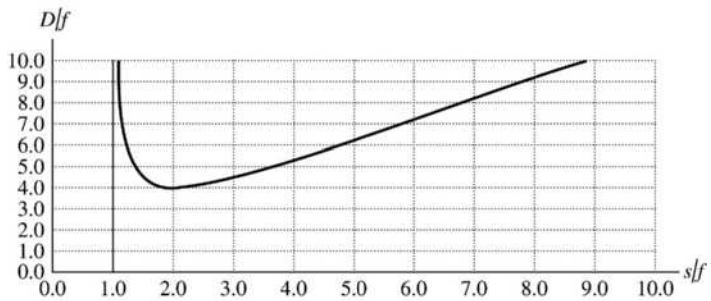


Figure 34.117

34.118. **IDENTIFY:** Use $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ to calculate s' (the distance of each point from the lens), for points A, B and C.

SET UP: The object and lens are shown in Figure 34.118a.

EXECUTE: **(a)** For point C: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{45.0\text{ cm}}+\frac{1}{s'}=\frac{1}{20.0\text{ cm}}\Rightarrow s'=36.0\text{ cm}.$

$y'=-\frac{s'}{s}y=-\frac{36.0}{45.0}(15.0\text{ cm})=-12.0\text{ cm},$ so the image of point C is 36.0 cm to the right of the lens, and 12.0 cm below the axis.

For point A: $s=45.0\text{ cm}+8.00\text{ cm}(\cos45°)=50.7\text{ cm}.$

$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{50.7\text{ cm}}+\frac{1}{s'}=\frac{1}{20.0\text{ cm}}\Rightarrow s'=33.0\text{ cm}.$

$y'==\frac{s'}{s}y=-\frac{33.0}{45.0}(15.0\text{ cm}-8.00\text{ cm}(\sin45°))=-6.10\text{ cm},$ so the image of point A is 33.0 cm to the right of the lens, and 6.10 cm below the axis.

For point B: $s=45.0\text{ cm}-8.00\text{ cm}(\cos45°)=39.3\text{ cm}.$

$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{39.3\text{ cm}}+\frac{1}{s'}=\frac{1}{20.0\text{ cm}}\Rightarrow s'=40.7\text{ cm}.$

$y'=-\frac{s'}{s}y=-\frac{40.7}{39.3}(15.0\text{ cm}+8.00\text{ cm}(\sin45°))=-21.4\text{ cm},$ so the image of point B is 40.7 cm to the right of the lens, and 21.4 cm below the axis. The image is shown in Figure 34.118b.

(b) The length of the pencil is the distance from point A to B:

$L=\sqrt{(x_A-x_B)^2+(y_A-y_B)^2}=\sqrt{(33.0\text{ cm}-40.7\text{ cm})^2+(6.10\text{ cm}-21.4\text{ cm})^2}=17.1\text{ cm}$

EVALUATE: The image is below the optic axis and is larger than the object.

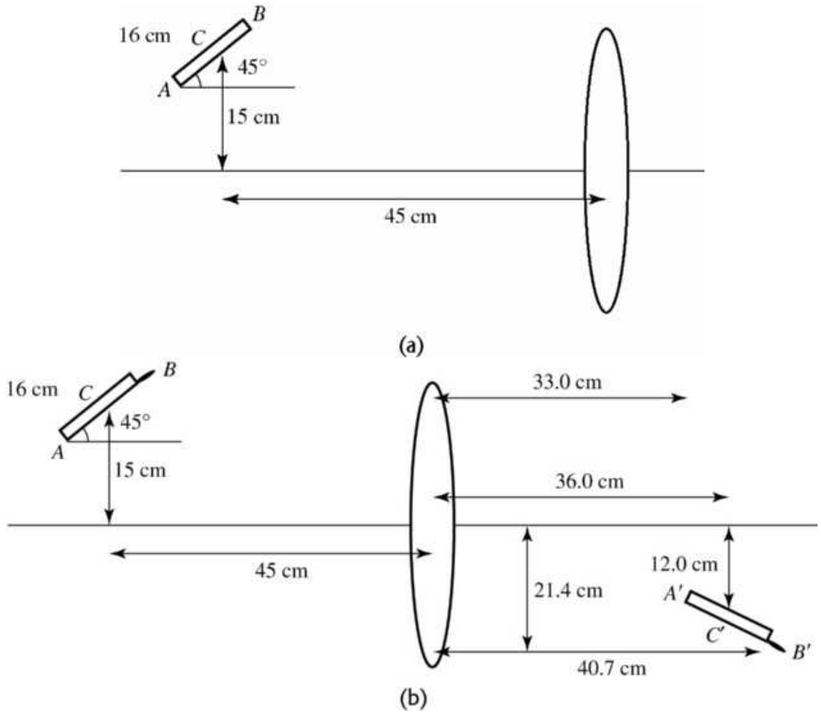


Figure 34.118

34.119. **IDENTIFY:** Apply $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$ to refraction at the cornea to find where the object for the cornea must be in order for the image to be at the retina. Then use $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ to calculate *f* so that the lens produces an image of a distant object at this point.

SET UP: For refraction at the cornea, $n_a=1.333$ and $n_b=1.40$. The distance from the cornea to the retina in this model of the eye is 2.60 cm. From Problem 34.52, $R=0.71$ cm.

EXECUTE: **(a)** People with normal vision cannot focus on distant objects under water because the image is unable to be focused in a short enough distance to form on the retina. Equivalently, the radius of curvature of the normal eye is about five or six times too great for focusing at the retina to occur.

(b) When introducing glasses, let's first consider what happens at the eye:

$\frac{n_a}{s_2}+\frac{n_b}{s_2'}=\frac{n_b-n_a}{R}\Rightarrow\frac{1.333}{s_2}+\frac{1.40}{2.6\text{ cm}}=\frac{0.067}{0.71\text{ cm}}\Rightarrow s_2=-3.00\text{ cm}.$ That is, the object for the cornea must be 3.00 cm behind the cornea. Now, assume the glasses are 2.00 cm in front of the eye, so $s_1'=2.00\text{ cm}+|s_2|=5.00\text{ cm}.$ $\frac{1}{s_1}+\frac{1}{s_1'}=\frac{1}{f_1'}$ gives $\frac{1}{\infty}+\frac{1}{5.00\text{ cm}}=\frac{1}{f_1'}$ and $f_1'=5.00$ cm. This is the focal length in water, but to get it in air, we use the formula from Problem 34.98:

$$f_1=f_1'\left[\frac{n-n_{\text{liq}}}{n_{\text{liq}}(n-1)}\right]=(5.00\text{ cm})\left[\frac{1.62-1.333}{1.333(1.62-1)}\right]=1.74\text{ cm}.$$

EVALUATE: A converging lens is needed.